MATHEMATICS

8

MODULE 4
ALGEBRA

Distance Learning

Alberta EDUCATION

Mathematics 8

Module 4: Algebra

MODULE BOOKLET

Mathematics 8
Student Module
Module 4
Algebra

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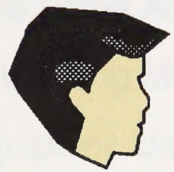
Welcome to Module 4!

We hope you'll enjoy your study of ***Algebra****.*

To make your learning a bit easier, a teacher will help guide you through the materials.

So whenever you see this icon,



turn on your audiocassette and listen.

CONTENTS AT A GLANCE

MODULE INTRODUCTION

What Lies Ahead

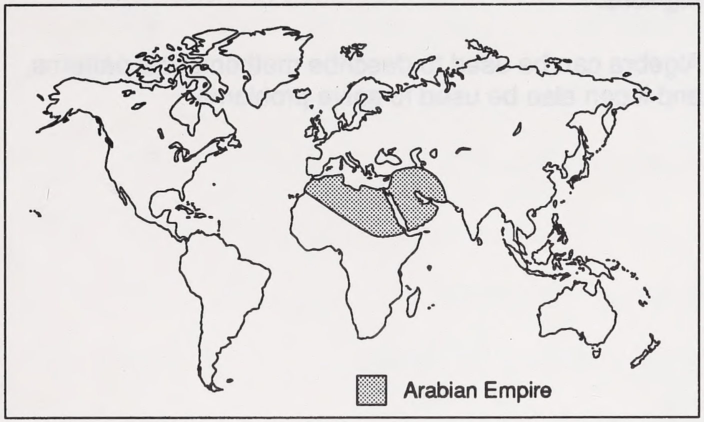
In the Module Introduction you will preview the module and learn how the module will be evaluated.

Working Together

In Module 4 of Mathematics 8 you will learn about **algebra**.

Algebra can be used to describe mathematical patterns, and it can also be used to solve problems.

You may be interested to learn that the study of algebra began in Baghdad around the eighth century. At that time the Arabian Empire stretched from Spain to India, and Baghdad was a cultural and scientific centre. Arabian scholars studied mathematics and developed algebra.



The English word *algebra* was borrowed from the Arabian word *al-jabr* which was used in the title of a book written by the Arabian mathematician, ibn-Musa Al-Khuwarizmi.

Another important person in the development of algebra was René Descartes (1596 - 1650).

Descartes was a French mathematician who developed the process of graphing algebraic relations using ordered pairs of numbers and describing these relations with equations.

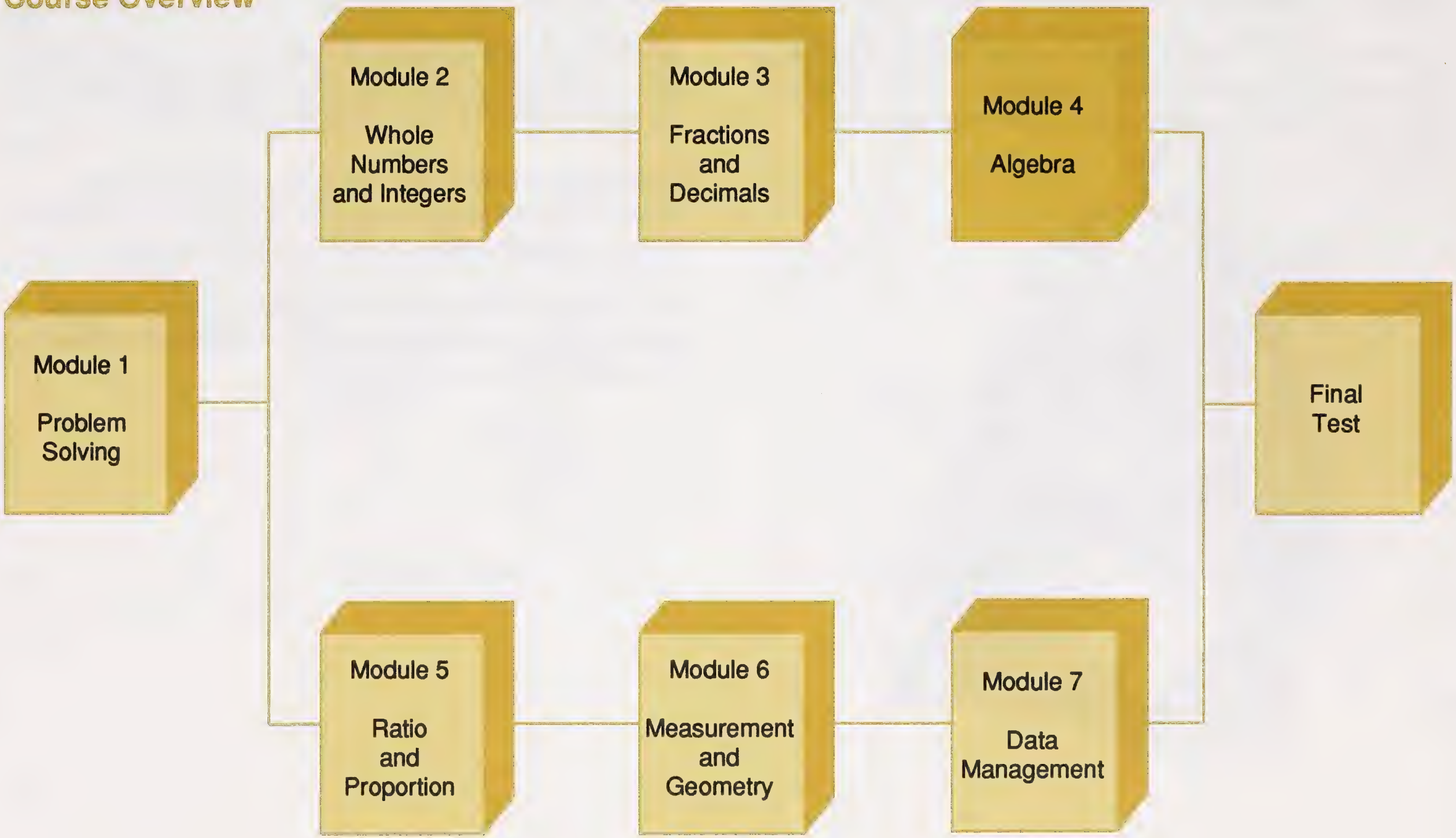
The story about how Descartes discovered coordinate geometry is an interesting one.

As a boy Descartes had poor health, so the rector of the Jesuit school he attended often allowed him to sleep in his room until he felt well enough to go to class.

During those mornings in his room, Descartes often amused himself by watching a fly on the ceiling and by trying to form an equation that described its path.

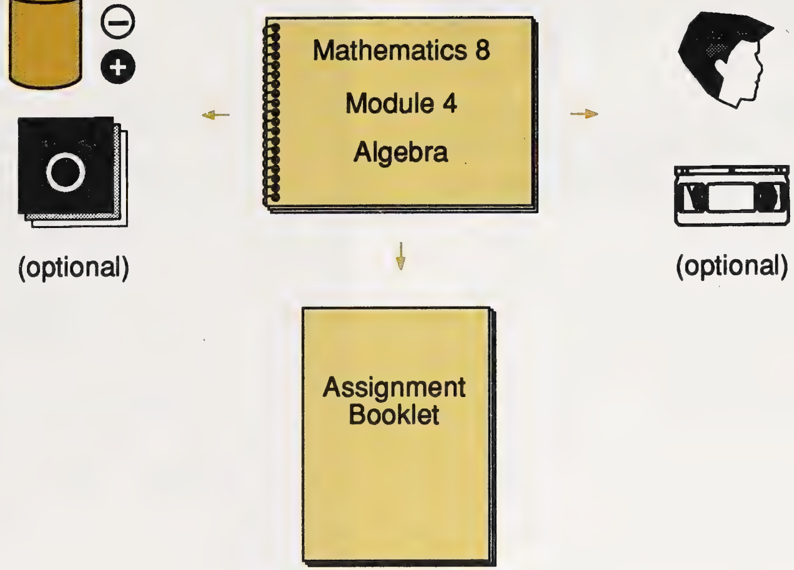
This practice led Descartes to develop the coordinate plane, which is sometimes called the **Cartesian plane** in recognition of his work.

Course Overview



Mathematics 8 has seven modules and a final supervised test. This module booklet is part of Module 4.

Module 4 Components



This module booklet will give you instruction and practice in learning mathematical skills and words. It will also direct you to the other components of the module. The computer and video activities in this booklet are optional; there are print alternatives. You should see your learning facilitator to check your answers to the activities in this module booklet. This module booklet is not to be submitted for a grade.

Your mark on this module will be determined by your work in the assignment booklet.

Take time to preview the module booklet before beginning Section 1.

What Lies Ahead

This section will pretest the following skills.

- using variables in an algebraic expression
- evaluating expressions for a given value of the variable
- simplifying algebraic expressions by combining like terms
- using variables to write mathematical expressions and sentences
- solving equations using additive inverses
- solving equations using multiplicative inverses
- solving equations using additive and multiplicative inverses
- generating a set of ordered pairs
- plotting ordered pairs
- producing a graph
- describing relations

Working Together

In Module 4 you will learn about algebra. The pretest in this section will help you and your learning facilitator determine your strengths and weaknesses.

Pretest

Space for Your Work

1. Translate each English phrase into a mathematical expression.

   a. five times Sam's age

   b. Helen's mass decreased by two kilograms

   c. the distance from John's house to work divided by two

   d. ten dollars more than Ruth's wage

   e. the sum of the length and width of the house

2. Use learning aids (cylinders and counters) to do the following.

   a. Model x and evaluate if $x = -1$.

   b. Model $-x$ and evaluate if $x = -1$.

   c. Model $x + 2$ and evaluate if $x = -1$.

   d. Model $x + y$ and evaluate if $x = -1$ and $y = +1$.

   e. Model $x + xy$ and evaluate if $x = -1$ and $y = +1$.

3. Use paper and pencil methods to evaluate the following expressions.

   a. $2a + 7$ if $a = 3$

   b. $n^2 - 2$ if $n = -5$

   c. $e - 3d$ if $d = 2$ and $e = -2$

   d. $3y + z$ if $y = \frac{1}{3}$ and $z = \frac{1}{2}$

4. Use learning aids (cylinders and counters) to model and simplify the following expressions.

   a. $t + t + t + t$

   b. $s + 2 - 3 + 1$

   c. $a + 2 + a - 5$

   d. $a + b - 3 + a$

   e. $b + c - 1 - b$

5. Use paper and pencil methods to simplify the following expressions.

   a. $3f - 1 + 2 - f$

   b. $2a + 3ab + a - b$

   c. $-cd + 3c^2 + cd - d$

   d. $2a - 2 + 5a$

Space for Your Work

6. Translate the following sentences into equations.

   a. Two times a number plus three results in thirteen.

   b. Linton's age is six less than Evon's.

   c. Katsuta's mass increased by 10 kg is the same as Hayanu's.

   d. The length of the building squared is five less than the width of the building.

7. Model the following equations. Then solve the equations using inspection or guess-check-revise methods. Verify the solutions.

   a. $x + 2 = 8$

   b. $3d = 12$

   c. $2m - 1 = 7$

   d. $5n = 2n - 9$

   e. $2y + 3 = y - 1$

Space for Your Work

8. Solve the following equations using inspection or guess-check-revise methods. Verify using paper and pencil methods.

   a. $3w + 2 = 11$

   b. $5x + 1 = 3x - 9$

9. Solve $x + 3 = 8$ using additive inverses. Verify the solution.

   a. Use models.

   b. Use paper and pencil methods.

10. Solve $3y = -12$ using multiplicative inverses. Verify the solution.

    a. Use models.

    b. Use paper and pencil methods.

11. Solve the following using paper and pencil methods. Verify the solutions.

    a. $\frac{r}{3} = 5$

    b. $\frac{n}{2} = \frac{7}{8}$

12. Solve $3d + 5 = 11$ using inverses. Verify the solution.

    a. Use models.

    b. Use paper and pencil methods.

Space for Your Work

13. Complete the following table of values.

Space for Your Work

$y = 3x - 5$	
x	y
-2	
-1	
0	
1	
2	

14. Graph $y = 3x - 5$.

15. Jerrit unpacks cans from boxes at a grocery store.

Space for Your Work

Number of Boxes (b)	Relation	Number of Cans (c)
1		
2		
3		
4		
5		

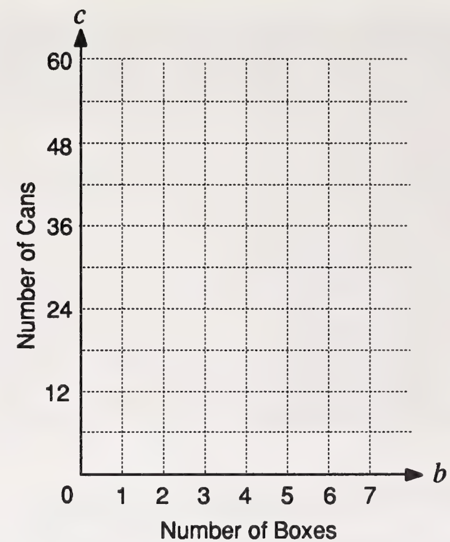
a. Use words to describe the relationship.

b. Write an equation to describe the relationship.

c. Use ordered pairs to describe the relationship.

d. Use a graph to describe the relationship.

Space for Your Work



See your learning facilitator to check your answers and to receive further instructions.

TRANSLATING WORDS AND PHRASES

What Lies Ahead

In this section you will learn these skills.

- translating English phrases into mathematical expressions
- interpreting the meaning of variables and algebraic expressions

Working Together

In algebra, English words and phrases can be translated into mathematical expressions with variables. Learning to translate English phrases into mathematical expressions is somewhat like learning a new language.

Translating Words and Phrases

Can you read each of these signs?

deux et trois	zwei und drei
two plus three	dos y tres

Different languages have been used, but all the signs say the same thing.

Each sign can be represented by this mathematical expression.

$$2 + 3$$

Mathematics is a common language understood by mathematicians all over the world, no matter what language they speak.

You should be aware that different English words or phrases can be translated into the same mathematical expression.

Example

- add two and three
- the sum of two and three
- two plus three
- two increased by three
- two more than three

Each of these statements can be translated into this mathematical expression.

$$2 + 3$$

When you translate from one language to another, you sometimes need to change the order of the words in the new language.

Example

A word-for-word English translation of the Spanish phrase *el perro blanco* would be *the dog white*, but you would say *the white dog* if you were using the rules of English grammar.

el perro blanco

the white dog

When you translate some English phrases into mathematics, you may also need to change the order to reflect the meaning.

Example

- two years more than three years

  3 + 2

- one nickel less than five nickels

  5 − 1

Note

Because of the commutative property for addition, you can translate *two years more than three years* as 3 + 2 or 2 + 3.

However, the commutative property does not hold for subtraction. Therefore, be careful when translating phrases involving subtraction. You must translate *one nickel less than five nickels* as 5 − 1.

Introductory Activities

Space for Your Work

Translate each English phrase into a mathematical expression

1. seven baseballs increased by nine baseballs

2. the difference between nine poodles and six poodles

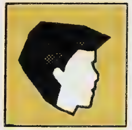
3. five groups of fifteen children

4. forty-eight pizzas divided by sixteen people

5. three more than the difference of five years and two years

6. thirty-two tapes decreased by eight groups of three tapes

✓ See your learning facilitator to check your answers and to receive further instructions.



Working Together

Now you will work with variables and algebraic expressions.

A **variable** is a symbol which replaces a number which is unknown.

An **algebraic expression** is a mathematical expression that uses one or more variables.

Sometimes geometrical shapes, such as circles, triangles, and squares, are used as variables.

Example: $\blacksquare + 3$

Here $\blacksquare$ is a variable. It is used as a place holder to represent an unknown number.

The word *variable* comes from the latin word *varius*, which means *various*.

There are various possibilities for the value of the variable. $\blacksquare$ could be 1, 2, 3, 4, or any other number.

$\blacksquare + 3$ is an algebraic expression. It means *a number plus three*.

Usually letters from the alphabet are used to represent variables.

Example: $n + 3$

Here n is used to replace an unknown number. However, any other lower case letter could be used instead of n.

The variable can be replaced by any type of number, such as a whole number, integer, fraction, or decimal.

$n + 3$ means *a number plus three*.

Many English phrases can be translated by using variables.

Example

Phrase	Variable
Sam's age	a
Helen's mass	m
the distance from John's house to work	d
Ruth's wages	w
the length of the house	ℓ
the width of the house	w

Notice that the variable is often the first letter of a word and lower case letters are usually selected.

Get in the habit of writing, rather than printing, variables.

These variables can then be used to translate other English phrases

Example

Phrase	Variable
five times Sam's age	$5 \times a$ or $5a$
Helen's mass decreased by two kilograms	$m - 2$
the distance from John's house to work divided by two	$d \div 2$ or $\frac{d}{2}$
ten dollars more than Ruth's wage	$w + 10$
the sum of the length and width of the house	$\ell + w$

Remember the following points when translating English phrases into algebraic expressions.

- Do not confuse the number 0 and the variable a.
- Do not confuse the number 1 and the variable ℓ.

- There are different ways to translate mathematical expressions involving multiplication.

Method 1

When translating expressions such as *five times Sam's age*, you usually would not use the multiplication sign between the number and the variable.

$5a$

If you do include the multiplication sign, be careful that it is not confused with the variable x.

$5 \times a$

times

When translating expressions such as *three times six*, you must use a multiplication sign between the numbers.

3×6

If you did not include the multiplication sign, it would look like this.

36

This would be read as *thirty-six*, not as *three times six*.

Method 2

Brackets can also be used to represent multiplication.

$(3)(6)$ $(5)(a)$

Method 3

Raised dots are sometimes used to show multiplication.

$3 \cdot 6$ $5 \cdot a$

If you use the raised dot, be careful that it is not mistaken as a decimal point.

- There are different ways to translate expressions involving division.

Method 1

When translating expressions like *the distance from John's house divided by two*, you may use the division sign between the numbers.

$d \div 2$

Method 2

You may also use the fraction form to represent division.

$\frac{d}{2}$

Practice Activities

Space for Your Work

1. Translate the following English expressions into mathematical expressions containing variables.

   a. the sum of a number and three

   b. three subtracted from a number

   c. two thirds of a number

   d. a number squared

   e. five more than a number

   f. twice a number increased by six

   g. one less than two thirds of a number

   h. four more than one half of a number

   i. the difference between eight and a number cubed

Print Alternative

Space for Your Work

2. Translate the following situations into mathematical phrases.

   a. Marco's age increased by two years

   b. twice the distance from Muriel's house to school

   c. a number squared

   d. five times the price of a car

   e. twice Amar's salary plus three dollars

   f. one half of the price

   g. a boy's age twelve years from now

   h. the sum of Ardith's mass and twenty-five kilograms

Computer Alternative

Space for Your Work

3. Do Introductory Lesson 1 of *BRITANNICA: Problem Solving in Algebra.*

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

1. Read the following phrases aloud.

   a. three times four, less two

   b. three times, four less two

   c. the sum of two, and five times three

   d. the sum of two and five, times three

2. How did you translate the commas when you read the phrases in Question 1?

3. Mathematicians sometimes use brackets to show order of operations. Translate the expressions in Question 1 into mathematical expressions containing brackets.

4. Translate the following English expressions into algebraic expressions.

   a. sixteen times, two more than a number

   b. sixteen times two, more than a number

   c. four times the length, plus eight centimetres

   d. four times, the length plus eight centimetres

5. Remember that subtraction is sometimes translated in a different order. Translate the following English expressions into mathematical expressions.

   a. the difference of three, and two times a number

   b. the difference of three and two, times a number

   c. four less than three, times a number

   d. four less than, three times a number

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

EVALUATING ALGEBRAIC EXPRESSIONS

What Lies Ahead

In this section you will learn these skills.

- modelling algebraic expressions
- evaluating algebraic expressions using models
- evaluating algebraic expressions using paper and pencil methods

In this section you will use these words.

- variables
- algebraic expressions
- evaluate
- substitute

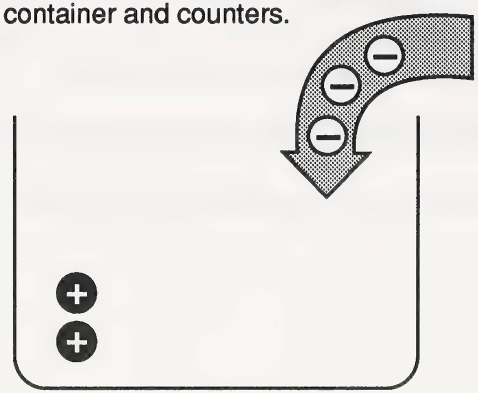
Working Together

In Module 2 you learned to model integers using counters.

Example

-  represents + 3.
-  represents – 2.
-  represents 0.

You also learned to model mathematical expressions using a container and counters.



This diagram represents (+ 2) + (– 3).

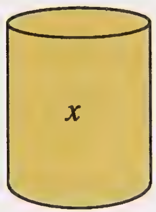
Modelling Variables

You can model variables in much the same way that you modelled integers. When modelling variables, you will use cylinders to represent the variables.

Example 1: How do you model x?

Solution

Model x this way.

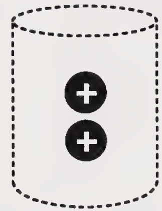


The cylinder represents an unknown number.

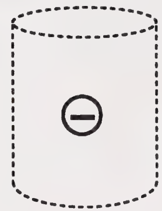
Evaluate x by replacing it with given numbers.

Evaluate means to find the value of the expression.

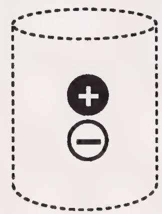
- If $x = +2$, replace the cylinder labelled x with $+2$.



- If $x = -1$, replace the cylinder labelled x with -1.



- If $x = 0$, replace the cylinder labelled x with 0.



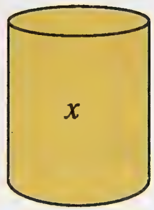
There are an infinite number of possibilities for the value of x.

Example 2: How do you model $-x$?

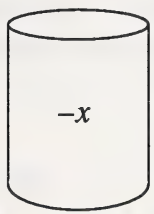
Solution

To model $-x$, use a cylinder that is the same size as the cylinder labelled x. This cylinder should be a different colour.

If x is modelled this way,



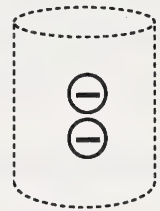
then $-x$ would be modelled this way.



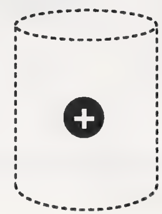
$-x$ means the opposite of x.

Evaluate $-x$ by replacing it with given numbers.

- If $x = +2$, then $-x = -2$. Replace the cylinder labelled $-x$ with -2.



- If $x = -1$, then $-x = +1$. Replace the cyclinder labelled $-x$ with $+1$.



There are an infinite number of possibilities for the value of $-x$.

Modelling Algebraic Expressions

You can also model and evaluate algebraic expressions with cylinders and counters.

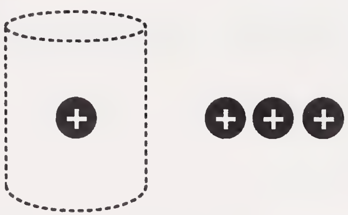
Example 1: How do you model and evaluate $d + 3$?

Solution

Use a cylinder and three positive counters to model $d + 3$.

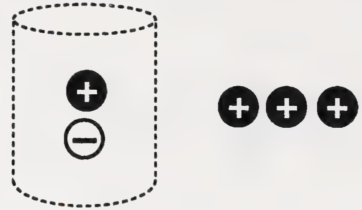
Evaluate $d + 3$ by replacing the variable with given numbers and finding the total.

- If $d = +1$, replace the cylinder labelled d with $+1$.



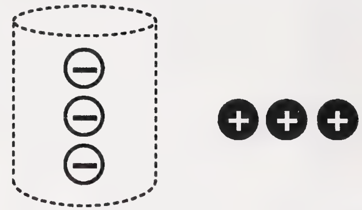
The value of the algebraic expression is + 4.

- If $d = 0$, replace the cylinder labelled d with 0.



The value of the algebraic expression is + 3.

- If $d = -3$, replace the cylinder labelled d with -3.



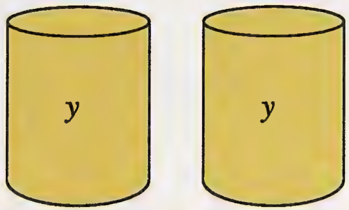
The value of the algebraic expression is 0.

There are an infinite number of possibilities for the value of d. The value of the algebraic expression depends on the value of the variable.

Example 2: How do you model and evaluate $2y$?

Solution

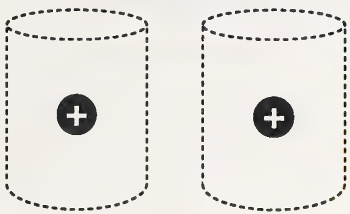
Use two cylinders to model $2y$.



Each cylinder is the same size and colour.

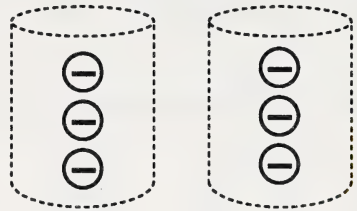
Evaluate $2y$ by replacing each cylinder labelled y with a given value.

- If $y = +1$, replace each cylinder labelled y with $+1$.



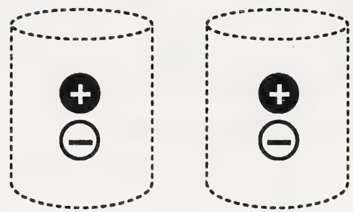
The value of the algebraic expression is $+2$.

- If $y = -3$, replace each cylinder labelled y with -3.



The value of the algebraic expression is -6.

- If $y = 0$, replace each cylinder labelled y with 0.

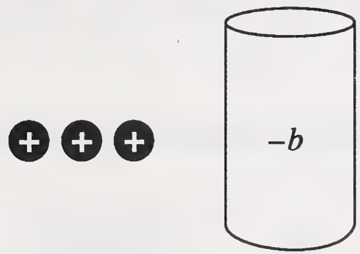


The value of the algebraic expression is 0.

Example 3: How do you model and evaluate $3 - b$?

Solution

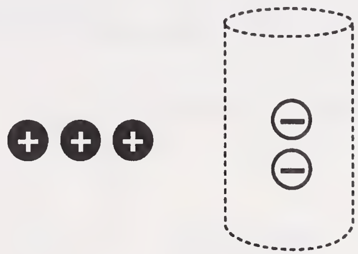
Represent $3 - b$ with a cylinder and three counters.



Evaluate $3 - b$ by replacing $-b$ with the opposite of b.

- If $b = +2$, then $-b = -2$.

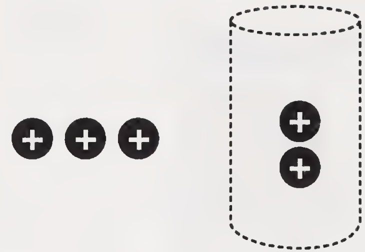
  So, replace the cylinder labelled $-b$ with -2.



The value of the algebraic expression is $+1$.

- If $b = -2$, then $-b = +2$.

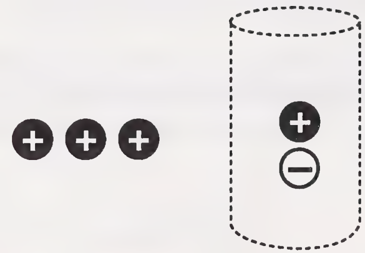
  So, replace the cylinder labelled $-b$ with $+2$.



The value of the algebraic expression is $+5$.

- If $b = 0$, then $-b = 0$.

  So, replace the cylinder labelled $-b$ with 0.

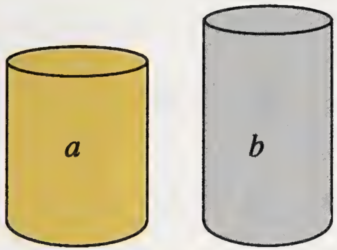


The value of the algebraic expression is $+3$.

Example 4: How do you model and evaluate $a + b$?

Solution

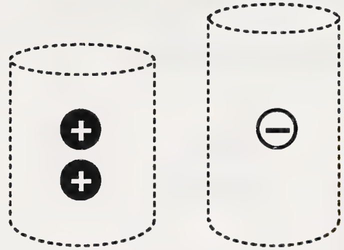
Represent $a + b$ by using two different-sized cylinders.



Each cylinder represents a different unknown.

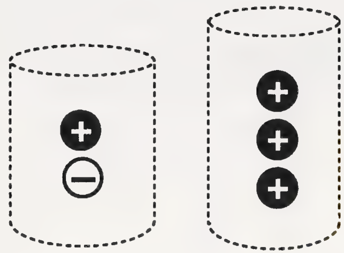
Evaluate $a + b$ by replacing the cylinder labelled a with a given value and the cylinder labelled b with another given value.

- If $a = +2$ and $b = -1$, replace the cylinder labelled a with $+2$ and the cylinder labelled b with -1.



The value of the algebraic expression is $+1$.

- If $a = 0$ and $b = +3$, replace the cylinder labelled a with 0 and the cylinder labelled b with $+3$.

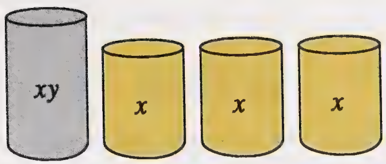


The value of the algebraic expression is $+3$.

Example 5: How do you model and evaluate $xy + 3x$?

Solution

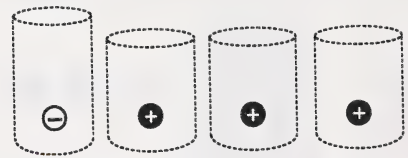
Represent $xy + 3x$ by using two different sizes of cylinders.



Evaluate $xy + 3x$ by replacing each of the cylinders labelled x with a given value and by replacing the cylinder labelled xy with the product of the given values for x and y.

- If $x = +1$ and $y = -1$, then $xy = (+1) \times (-1)$ or -1.

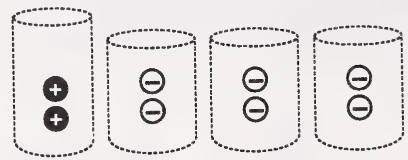
  So, replace each cylinder labelled x with $+1$ and the cylinder labelled xy with -1.



The value of the algebraic expression is $+2$.

- If $x = -2$ and $y = -1$, then $xy = (-2) \times (-1)$ or $+2$.

  So, replace each cylinder labelled x with -2 and the cylinder labelled xy with $+2$.

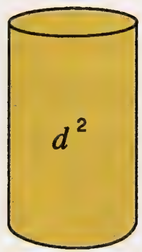


The value of the algebraic expression is -4.

Example 6: How do you model and evaluate d^2?

Solution

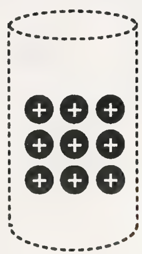
Model d^2 this way.



Evaluate d^2 by replacing the cylinder labelled d^2 with the square of the given value of d.

- If $d = +3$, then $d^2 = (+3)^2$ or $+9$.

  So, replace the cylinder labelled d^2 with $+9$.



The value of the algebraic expression is $+9$.

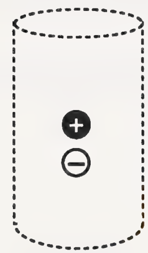
- If $d = -2$, then $d^2 = (-2)^2$ or $+4$.

  So, replace the cylinder labelled d^2 with $+4$.

The value of the algebraic expression is $+4$.

- If $d = 0$, then $d^2 = 0^2$ or 0.

  So, replace the cylinder labelled d^2 with 0.

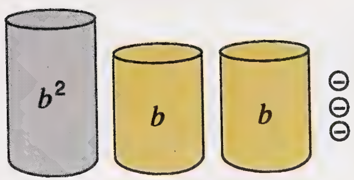


The value of the algebraic expression is 0.

Example 7: How do you model and evaluate $b^2 + 2b - 3$?

Solution

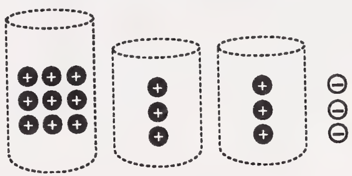
Model the algebraic expression this way.



Evaluate $b^2 + 2b - 3$ this way.

- If $b = +3$, then $b^2 = (+3)^2$ or 9.

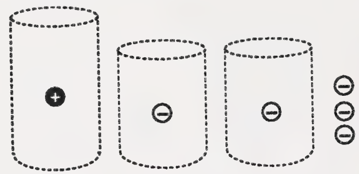
  So, replace the cylinders labelled b with + 3 and the cylinder labelled b^2 with + 9.



The value of the expression is + 12.

- If $b = -1$, then $b^2 = (-1)^2$ or + 1.

  So, replace the cylinders labelled b with – 1 and the cylinder labelled b^2 with + 1.



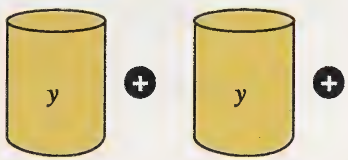
The value of the expression is – 4.

Example 8: How do you model and evaluate $2(y + 1)$?

$2(y + 1)$ means two groups of $y + 1$.

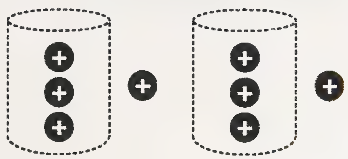
Solution

Model $2(y + 1)$ this way.



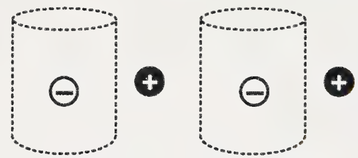
Evaluate $2(y + 1)$ this way.

- If $y = +3$, replace each cylinder labelled y with $+3$.



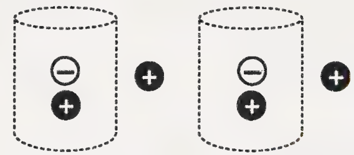
The value of the algebraic expression is + 8.

- If $y = -1$, replace each cylinder labelled y with -1.



The value of the algebraic expression is 0.

- If $y = 0$, replace each cyinder labelled y with 0.



The value of the algebraic expression is + 2.

Introductory Activities

Space for Your Work

1. a. Model $x + 5$.

   b. Evaluate $x + 5$ if $x = +3$.

   c. Evaluate $x + 5$ if $x = -2$.

2. a. Model $3 - y$.

   b. Evaluate $3 - y$ if $y = +4$.

   c. Evaluate $3 - y$ if $y = -1$.

3. a. Model $3z - 5$.

   b. Evaluate $3z - 5$ if $z = -2$.

   c. Evaluate $3z - 5$ if $z = +3$.

4. a. Model $a + 2b$.

   b. Evaluate $a + 2b$ if $a = -1$ and $b = +3$.

   c. Evaluate $a + 2b$ if $a = +2$ and $b = -2$.

5. a. Model $m^2 + n^2$.

   b. Evaluate $m^2 + n^2$ if $m = +2$ and $n = +3$.

   c. Evaluate $m^2 + n^2$ if $m = -1$ and $n = 0$.

6. a. Model $r + 2rs$.

   b. Evaluate $r + 2rs$ if $r = -1$ and $s = +2$.

   c. Evaluate $r + 2rs$ if $r = +3$ and $s = -1$.

7. a. Model $3(a - 5)$.

   b. Evaluate $3(a - 5)$ if $a = +1$.

   c. Evaluate $3(a - 5)$ if $a = -1$.

Space for Your Work

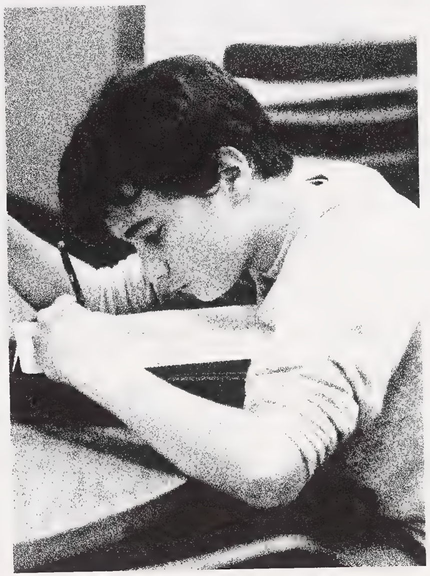
See your learning facilitator to check your answers and to receive further instructions.

Working Together

Evaluating Expressions

In the Introductory Activities you evaluated algebraic expressions using models.

Now you will learn to evaluate algebraic expressions with paper and pencil methods.



Example 1

Evaluate $a + 7$ if $a = 10$.

Solution

$$\begin{aligned} a + 7 &= 10 + 7 \\ &= 17 \end{aligned}$$

Example 2

Evaluate $n^2 - 2$ if $n = -5$.

Solution

$$\begin{aligned} n^2 - 2 &= (-5)^2 - 2 \\ &= 25 - 2 \\ &= 23 \end{aligned}$$

Example 3

Evaluate $e - 3d$ if $d = 5$ and $e = 70$.

Solution

$$\begin{aligned} e - 3d &= 70 - 3 \times 5 \\ &= 70 - 15 \\ &= 55 \end{aligned}$$

Use the rules for order of operations. Multiplication is completed before subtraction.

Example 4

Evaluate $3y + z$ if $y = \frac{1}{2}$ and $z = \frac{5}{6}$.

Solution

$$\begin{aligned} 3y + z &= 3 \times \frac{1}{2} + \frac{5}{6} \\ &= \frac{3}{2} + \frac{5}{6} \\ &= \frac{9}{6} + \frac{5}{6} \\ &= \frac{14}{6} \\ &= 2\frac{2}{6} \\ &= 2\frac{1}{3} \end{aligned}$$

Use common denominators.

Example 5

Evaluate $3xy$ if $x = 0.5$ and $y = 1.5$.

Solution

$$\begin{aligned} 3xy &= 3 \times 0.5 \times 1.5 \\ &= 2.25 \end{aligned}$$

Example 6

Evaluate $a^2 - 2ab - b^2$ if $a = +2$ and $b = -1$.

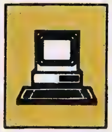
Solution

$$\begin{aligned} a^2 - 2ab - b^2 &= (+2)^2 - 2 \times (+2) \times (-1) - (-1)^2 \\ &= (+4) - (-4) - (+1) \\ &= (+4) + (+4) + (-1) \\ &= +7 \end{aligned}$$

Practice Activities

Space for Your Work

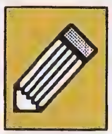
Computer Alternative



1. Do Lesson 5 of the *Pre-Algebra* disk from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

   Remember, if you need help, press the SHIFT key and the ? key.

Print Alternative



2. a. Evaluate $n + 10$ if $n = 8$.

   b. Evaluate $2p$ if $p = 0.6$.

   c. Evaluate $5r - 2$ if $r = \frac{3}{5}$.

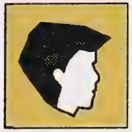
d. Evaluate $3 + 4s$ if $s = 0.5$.

Space for Your Work

e. Evaluate $2n^2 + 1$ if $n = 4$.

f. Evaluate $0.5n - 0.1$ if $n = 2$.

✓ See your learning facilitator to check your answers and to receive further instructions.



Working Together

Sometimes it is necessary to evaluate an algebraic expression for several values of the variable. In these cases a table is helpful.

Example: Evaluate $a + 5$ if $a = 1, 2, 3, 4, 5$, and 6.

Solution

Evaluate by substituting the first three values.

- If $a = 1$, then $a + 5$
  $= 1 + 5$
  $= 6$
- If $a = 2$, then $a + 5$
  $= 2 + 5$
  $= 7$
- If $a = 3$, then $a + 5$
  $= 3 + 5$
  $= 8$

Then organize the values in a table to discover a pattern.

a	$a + 5$	Pattern
1	6	
2	7	$+1$
3	8	$+1$

Apply the pattern to find the last three values.

a	$a + 5$	Pattern
1	6	
2	7	$+1$
3	8	$+1$
4	9	$+1$
5	10	$+1$
6	11	$+1$

Concluding Activities

Space for Your Work

1. Complete the following tables. Evaluate the first three values and then use a pattern to help you find the last three values.

a.

a	$a + 2$
1	
2	
3	
4	
5	
6	

b.

b	$4b - 1$
1	
2	
3	
4	
5	
6	

c.

c	$2c$
1	
2	
3	
4	
5	
6	

Space for Your Work

d.

d	$3d + 2$
1	
2	
3	
4	
5	
6	

e.

f	$5f - 1$
1	
2	
3	
4	
5	
6	

Space for Your Work

2. How are the patterns in Question 1 similar to the algebraic expressions?
Hint: $a + 2 = 1a + 2$.

3. Each of the following algebraic expressions has a missing number shown by ▩ . Use the table of values to find each value of ▩ .

a.

k	▩$k - 3$
1	1
2	5
3	9
4	13

Space for Your Work

b.

y	▩ $y + 1$
1	7
2	13
3	19
4	25

c.

d	▩ $d - 1$
1	1
2	3
3	5
4	7

d.

x	▩ $x + 4$
1	7
2	10
3	13
4	16

4. Complete the following tables. Evaluate the first three variables and then use a pattern to help you find the last three values.

Space for Your Work

a.

a	a^2
1	
2	
3	
4	
5	
6	

b.

b	$b^2 + 3$
1	
2	
3	
4	
5	
6	

c.

c	$c^2 - 1$
1	
2	
3	
4	
5	
6	

Space for Your Work

5. Complete the following statements.

   a. In Question 1 the exponent of each variable is ______, and you found the difference ______ to obtain the pattern. Hint: $a + 2 = a^1 + 2$ and $4b - 1 = 4b^1 - 1$.

   b. In Question 4 the exponent of each variable is ______, and you found the difference ______ to obtain the pattern.

6. Each of the following algebraic expressions has the exponent of the variable missing. Use the table of values to find the missing exponent.

Space for Your Work

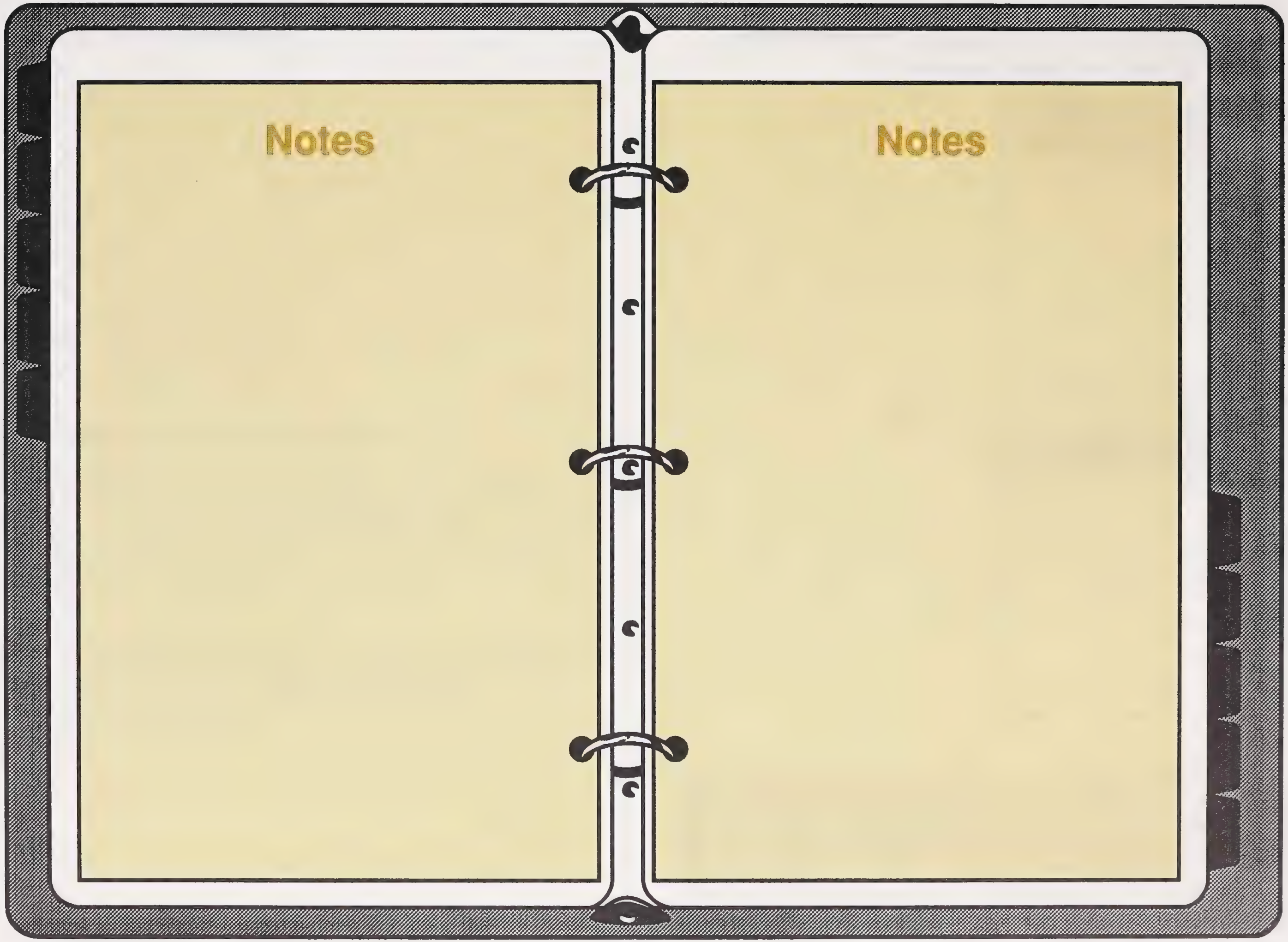
a.

a	$a^{\blacksquare} + 1$
1	2
2	5
3	10
4	17

b.

b	$b^{\blacksquare} - 1$
1	0
2	1
3	2
4	3

See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

SIMPLIFYING ALGEBRAIC EXPRESSIONS

What Lies Ahead

In this section you will learn these skills.

- identifying like terms and unlike terms
- simplifying algebraic expressions using models
- simplifying algebraic expressions using paper and pencil methods

In this section you will use these words.

- terms
- like terms
- unlike terms

Working Together

Mathematicians prefer to work with the simplest form of a number.

Example

$$\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$$

$\frac{1}{2}$ is the simplest form of $\frac{4}{8}$.

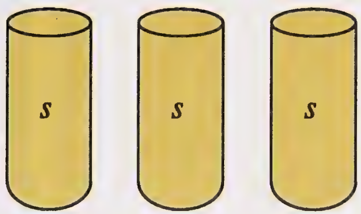
Some algebraic expressions may be simplified, too.

Example 1

How can you simplify $s + s + s$?

Solution

Model $s + s + s$.



From the model you can see that there are three groups of s.

So, $s + s + s = 3 \times s$ or $3s$.

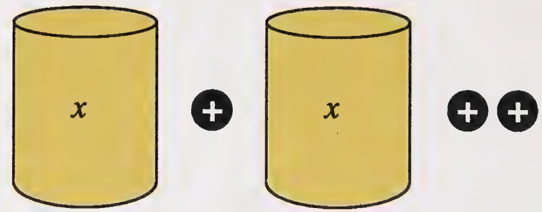
The multiplication sign is understood in $3s$.

Example 2

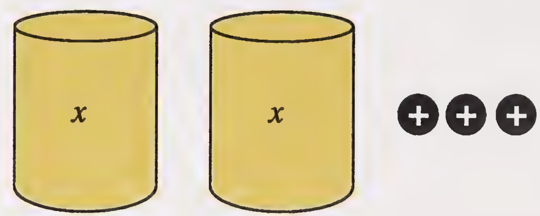
How can you simplify $x + 1 + x + 2$?

Solution

First model $x + 1 + x + 2$.



Then group the like objects.



From the model you can see that there are two groups of x.

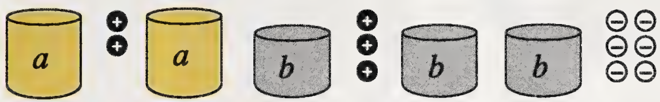
So, $x + 1 + x + 2 = 2x + 3$.

Example 3

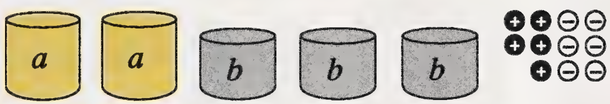
How can you simplify $a + 2 + a + b + 3 + b + b - 6$?

Solution

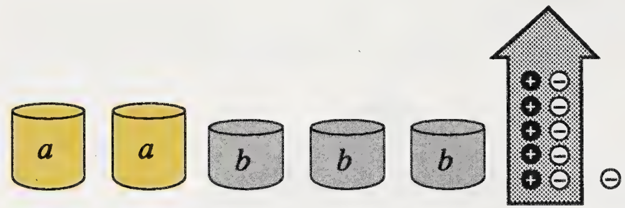
First model the expression.



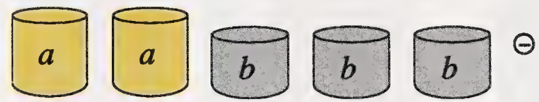
Then group the like terms.



Simplify by removing zeros. This does not change the value.



The result is this.



So, the simplest form is $2a + 3b - 1$.

Example 4

How can you simplify $x + 3 + x + 3$?

Solution

Method 1

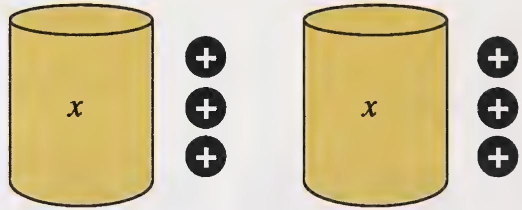
First model the expression.

Notice that there are two groups of $x + 3$.

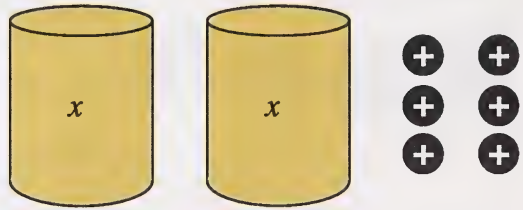
This can be expressed as $2(x + 3)$.

Method 2

First model the expression.



Then group the like objects.



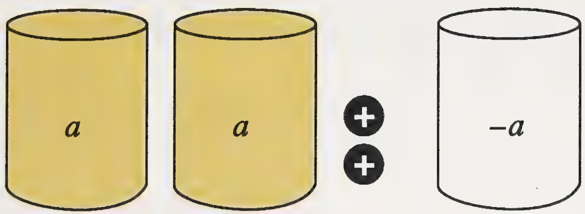
The simplest form is $2x + 6$.

Example 5

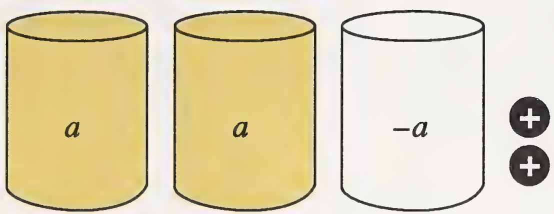
How can you simplify $a + a + 2 - a$?

Solution

First model the expression.

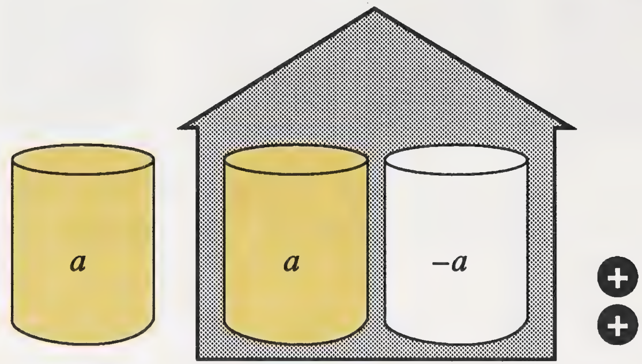


Then group the like objects.

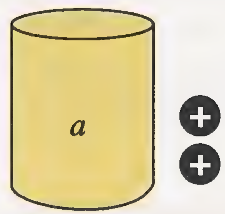


Because a and $-a$ are opposites, together they result in zero.

Simplify by removing the zeros. This does not change the value of the algebraic expression.



This is the result.



So, the algebraic expression in simplest form is $a + 2$.

Introductory Activities

Space for Your Work

Use models to simplify the following expressions.

1. $m + m$

2. $d + 2 + d - 4$

3. $y + y + 3 + x + 1 + x - 6 + x$

4. $a + y + a - 3$

5. $-4 + n + n + 7 - 6 + n$

6. $j + 3 + j + 3 + j + 3$

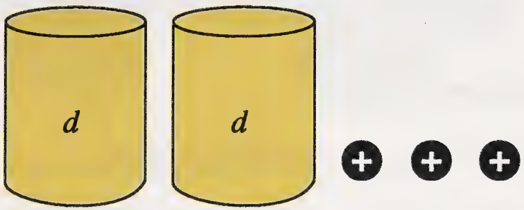
See your learning facilitator to check your answers and to receive further instructions.

Working Together

Now that you have simplified algebraic expressions using models, consider these examples.

Example 1

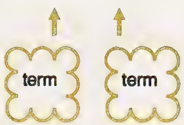
This model has two identical cylinders and three counters.



This model represents $2d + 3$.

The expression has two different terms.

$2d + 3$

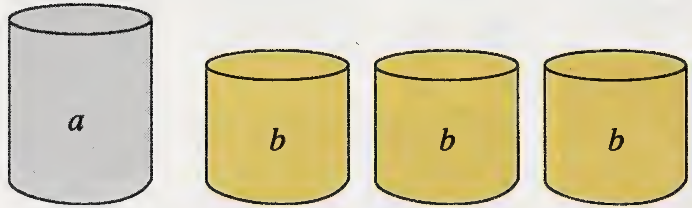


- The term $2d$ means $2 \times d$. It has a numerical factor and a variable factor.
- The term 3 means $3 \times (+1)$. It has numerical factors.

A **term** is the product of numerical factors and/or variable factors.

Example 2

This model has two different sizes of cylinders.



This model represents $a + 3b$.

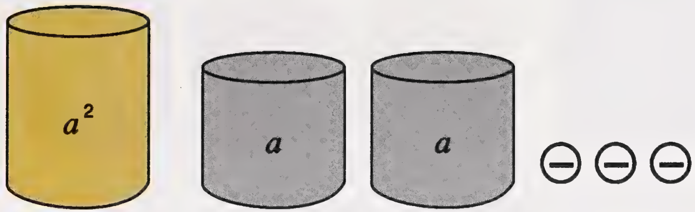
The expression $a + 3b$ has two different terms.

$a + 3b$

- The term a means $1 \times a$. It has a numerical factor and a variable factor.
- The term $3b$ means $3 \times b$. It has a numerical factor and a variable factor.

Example 3

This model has two different sizes of cylinders and three counters.



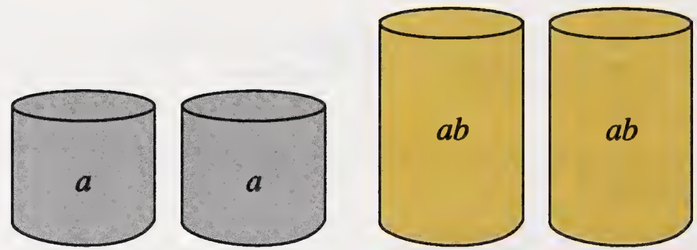
This model represents $a^2 + 2a - 3$.

The expression $a^2 + 2a - 3$ has three different terms.

- The term a^2 means $a \times a$. It has variable factors.
- The term $2a$ means $2 \times a$. It has a numerical factor and a variable factor.
- The term -3 means $3 \times (-1)$. It has numerical factors.

Example 4

This model has two different sizes of cylinders.



This model represents $2a + 2ab$.

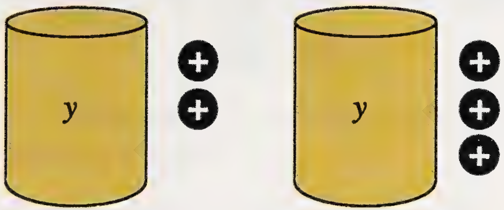
The expression $2a + 2ab$ has two different terms.

- The term $2a$ means $2 \times a$. It has a numerical factor and a variable factor.
- The term $2ab$ means $2 \times a \times b$. It has a numerical factor and two variable factors.

Like Terms

In the Introductory Activities you learned to simplify algebraic expressions.

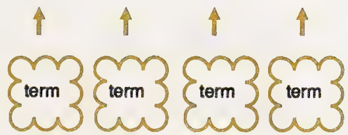
Example 1



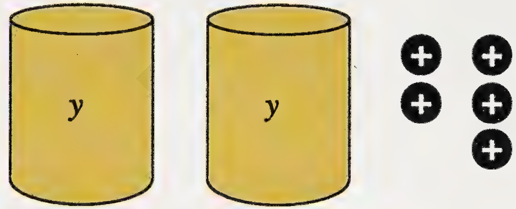
This model represents the expression $y + 2 + y + 3$.

The expression has four terms.

$$y \quad + \quad 2 \quad + \quad y \quad + \quad 3$$



You can simplify by combining the **like terms**.



So, $y + 2 + y + 3 = 2y + 5$

This is called **combining like terms**.

Note

You cannot combine $2y$ and $+ 5$. They are **unlike terms**.

You can combine like terms using paper and pencil methods.

Example 1

Simplify $2a + 3a$.

Solution

$$2a + 3a = 5a$$

The terms can be added because they are alike.

Example 2

Simplify $17c - 7c$.

Solution

$$17c - 7c = 10c$$

You can subtract because the terms are alike.

Example 3

Simplify $3a + 8n + 4 + 5a - 2n$.

Solution

$$3a + 8n + 4 + 5a - 2n = 3a + 5a + 8n - 2n + 4$$
$$= 8a + 6n + 4$$

You cannot combine $8a$, $6n$, and $+4$. They are unlike terms.

Example 4

Simplify $3x - 2xy + x + x^2$.

Solution

$$3x - 2xy + x + x^2 = x^2 + 3x + x - 2xy$$
$$= x^2 + 4x - 2xy$$

You cannot combine x^2, $4x$, and $-2xy$. They are unlike terms.

Practice Activities

Space for Your Work

Computer Alternative

1. Do Lesson 9 of the *Pre-Algebra* disk from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

Print Alternative

2. Simplify by collecting the like terms

   a. $2f + 5f$

   b. $5x - 2x + 4z - 2z$

   c. $3a - 2a^2 - 5a$

   d. $3m - 2 + 4p + 7m + 7n - 13$

   e. $2a + 3b + 2ab + 5b - 8ab$

   f. $-6cd + 3c^2 + 2d^2 + cd + 4d^2$

Extra Practice

Space for Your Work

1. Use the words **like** and **unlike** to describe each of the following pairs.

   a. $3x$ and x

   b. $4y$ and $4x$

   c. $-2a^2$ and a

   d. 8 and $\frac{1}{5}$

2. Simplify the following expressions.

   a. $6 + 4p - 3 - 6p$

   b. $3a - a^2 + 5a - 4a^2$

   c. $6a - 4ab + ab - 7a + 8a$

   d. $-4p + 6n - 3n + 7p - 6p + 5n$

   e. $4w - 7 + 3x + 5w + 13x - wx$

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

1. Which of the following expressions are equivalent?

   a. $a^2 - b^2 + 2b^2 - a^2$

   b. $2a^2 + 3a - 2a^2$

   c. $3a - b + a + 2b$

   d. $-3b + 3a + 4b - 2a$

   e. $2a - b - a + 2b$

   f. $6a - 8 + 3a + 5$

   g. $3a + a + 4b - 3b$

   h. $2ab - b^2 - 2ab + 2b^2$

   i. $4a + 2b - a - 2b$

   j. $a^2 + a - a^2 + 2a$

2. Evaluate the following.

   a. $2m + 6m$, if $m = 2$

   b. $5q - 2q$, if $q = 2.7$

   c. $5t - 3t + 2t$, if $t = \frac{1}{4}$

   d. $3f - g - f + g$, if $f = 2$ and $g = 5$

3. Collect the like terms in Question 2. Then evaluate the simplified expressions.

4. What do you notice about the answers in Questions 2 and 3?

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

TRANSLATING SENTENCES

What Lies Ahead

In this section you will learn this skill.

- translating English sentences into equations

Working Together

In Section 4 you learned that English phrases can be written as algebraic expressions. This skill can be extended to translating complete sentences into equations. The equations can then be used to solve problems.

Translating Sentences

Some English sentences can be translated into equations with one or more variables.

First you have to determine the variable.

Phrase	Variable
a number	n
the length	ℓ
Arthur's mass	a
Beth's mass	b
Raju's age	r
Jack's age	j

Once you have determined the variable, you can write an algebraic equation.

Sentence	Equation
A number increased by six gives thirteen.	$n + 6 = 13$
The length decreased by 4 cm results in 25 cm.	$\ell - 4 = 25$
Arthur's mass is the same as Beth's mass increased by 10 kg.	$a = b + 10$ or $a - b = 10$ or $b = a - 10$
Raju's age is twice Jack's age.	$r = 2j$ or $j = \frac{1}{2}r$

Notice that *gives*, *results in*, *is the same as*, and *is* can all be translated by an equals sign (=).

Introductory Activities

Space for Your Work

Print Alternative

1. Write each of the following sentences as an equation.
   a. Twelve decreased by a number is four.
   b. The sum of the number and one half of the number is equal to forty-eight.
   c. A number tripled decreased by 2 results in – 7.
   d. Three kilograms more than Bob's mass is fifty-two kilograms.
   e. Five dollars more than double Marnie's money is $79.
   f. Nine less than half the total number of newspapers delivered is twenty-seven newspapers.

Computer Alternative

2. Do Introductory Lessons 5 and 6 of *BRITANNICA: Problem Solving in Algebra.*

See your learning facilitator to check your answers and to receive further instructions.

Practice Activities

Space for Your Work

Write an equation for each of these problems.

1. Six times a number plus ten equals forty. What is the number?

2. A number increased by two and a half equals seven. What is the number?

3. The price of six hockey tickets plus eight dollars for refreshments equals eighty dollars. How much did each hockey ticket cost?

4. Chandia used an average of six sheets of loose-leaf paper each day. At the end of the semester she had used six hundred fifty-four sheets of paper. How many days long was the semester?

5. Zaib now has thirty-seven tropical fish in his aquarium. This is seven less than four times the number of fish that he had when he started his aquarium. How many tropical fish did Zaib have at the start?

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

Translate the following. Use brackets where needed.

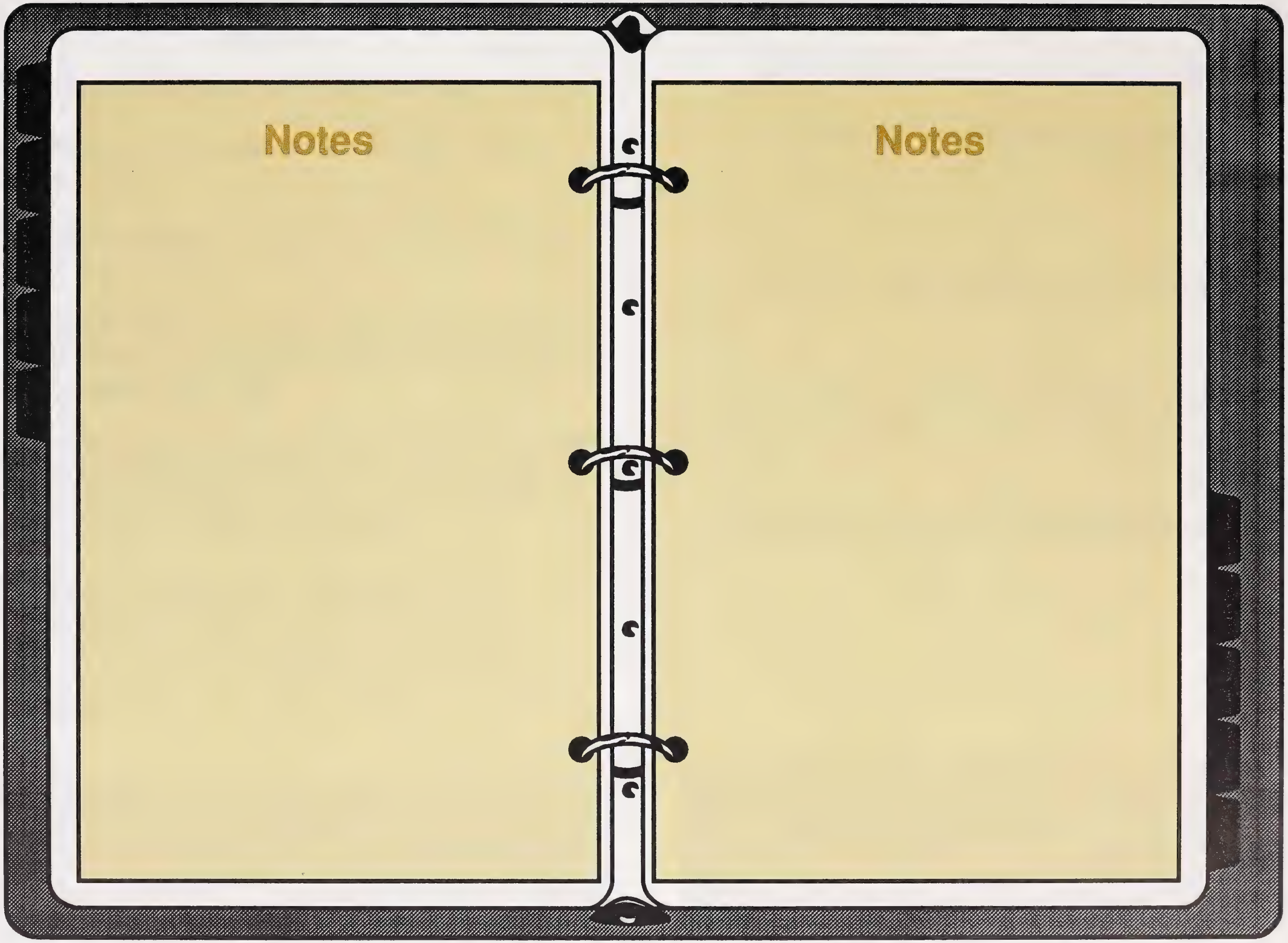
1. The square of nine, minus three, results in seventy-eight.

2. The square of, nine minus three, gives thirty-six.

3. The sum of eight and two, times a number is sixty.

4. The sum of eight, and two times a number equals twenty.

✓ See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

EQUATIONS

What Lies Ahead

In this section you will learn these skills.

- modelling equations
- solving equations by inspection or by using guess-check-revise methods
- verifying solutions by modelling the equation

In this section you will learn these words.

- equation
- balance

Working Together

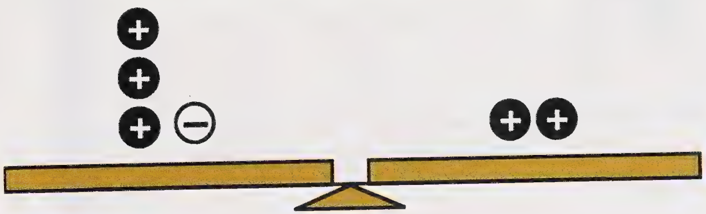
In Section 2 you used models to represent algebraic expressions. In this section you will model algebraic equations.

An **equation** is any mathematical statement that contains an equals sign (=). For an equation to be true, the value of each side of the equation must be the same. For this reason, a balance scale is sometimes used when modelling equations.

Video Activity

Watch the first segment on the video *MATH MOVES: Equations — Solving With One Step* and read the notes that follow. If you cannot watch the video, simply read the notes.

Here is an example of an **equation**.



The left-hand side (LS) of the scale has a value of + 2.
(Remember that ⊕ ⊖ means 0.)

The right-hand side (RS) of the scale also has a value of + 2.

Therefore, the LS and RS are balanced.

This situation can be described by this equation.

$+2 = +2$

= means *is equal to*.

Here is an example of an **inequation**.

The LS of the scale has a value of – 1.

The RS has a value of + 1.

The LS and RS are not balanced.

This situation can be described by this inequation.

$-1 \neq +1$

≠ means *is not equal to*.

This situation can also be described in these ways.

$-1 < +1$ or $+1 > -1$

< means *is less than*.

> means *is greater than*.

Some equations contain variables. These equations are called **algebraic equations** or **conditional equations**. These algebraic equations are only true for certain values of the variables.

Example 1

This model represents the equation $r + 2 = 3$.

How would you solve the equation $r + 2 = 3$? **Solving the equation** means finding the value of the variable which makes each side of the equation balanced.

By looking at the scale, you can see that $r = +1$ makes the equation true. $r = +1$ is called the **solution**. The solution is the value of the variable which makes the equation a true statement.

How can you verify the solution? **Verifying the solution** means testing to see if the solution gives a true statement.

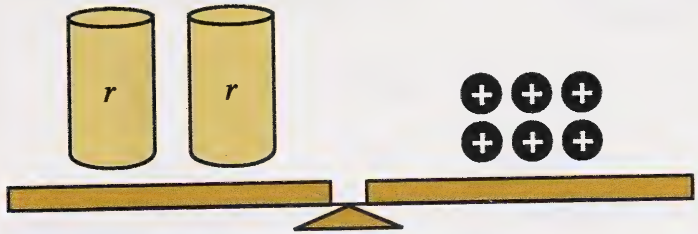
To check, replace the r with $+1$.

Each side of the scale has a value of $+3$ if $r = +1$.

The scale is balanced.

So, $r = +1$ makes the equation a true statement.

Example 2



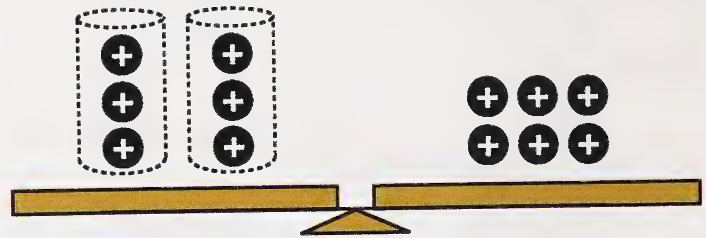
This model represents the equation $2r = 6$.

How would you solve the equation?

By looking at the scale, you can see that $r = +3$ makes the equation true.

How would you check or verify the solution?

Replace r with $+3$.

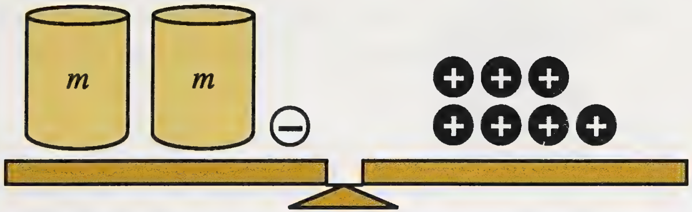


Each side of the scale has a value of $+6$ if $r = +3$.

The scale is balanced.

So, $r = +3$ makes the equation true.

Example 3



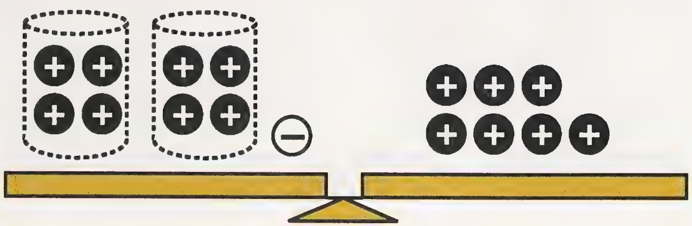
This model represents the equation $2m - 1 = 7$.

How would you solve the equation?

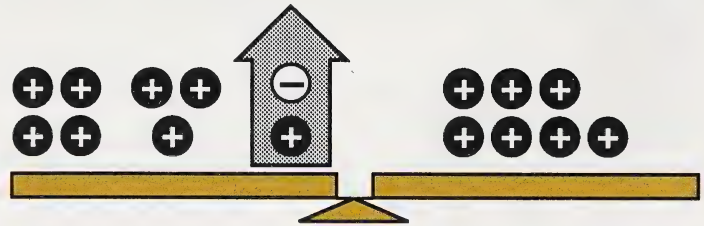
By looking at the scale, you can see that $m = +4$ makes the equation true.

How can you check or verify the solution?

First replace m with $+4$.



Then simplify by removing the zeros.



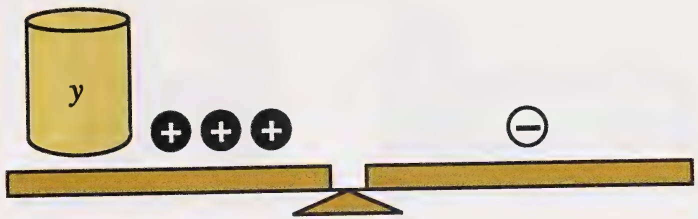
The simplified scale looks like this.

Each side of the scale has a value of $+7$ if $m = +4$.

The scale is balanced.

So, $m = +4$.

Example 4



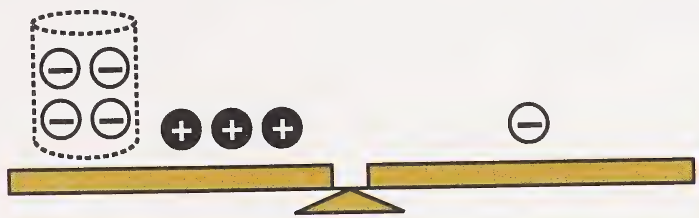
This model represents the equation $y + 3 = -1$.

How can you solve the equation?

By looking at the scale, you can see that $y = -4$ makes the equation true.

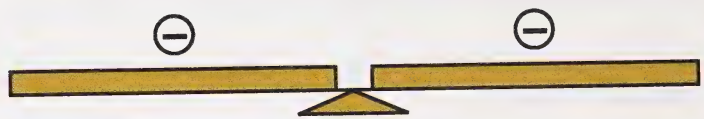
How can you check or verify the solution?

First replace y with -4.



Then simplify the equation by removing the zeros.

The simplified scale looks like this.



Each side of the scale has a value of -1 if $y = -4$.

The scale is balanced.

So, $y = -4$.

In the first part of this section you were able to solve equations by inspection. Sometimes this is not possible, and you must use the guess-check-revise method. This method is sometimes called guess and test.

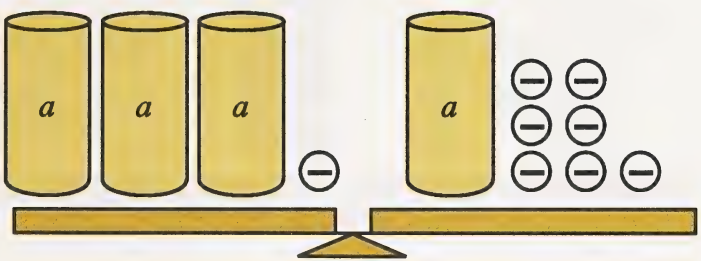
Video Activity

Watch the second segment on the video *MATH MOVES: Equations — Solving With One Step* and read the notes that follow. If you cannot watch the video, simply read the notes.

Example 1

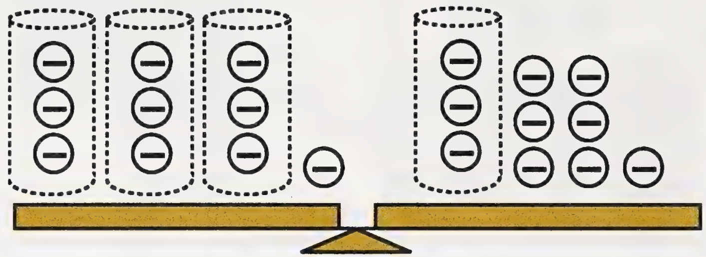
Solve $3a - 1 = a - 7$.

You can use a balance to model the equation.



To solve $3a - 1 = a - 7$ you may have to guess, check, and revise several times before you discover that $a = -3$ is the solution.

Verify this by replacing each a with -3.



Each side of the scale has a value of -10 if $a = -3$.

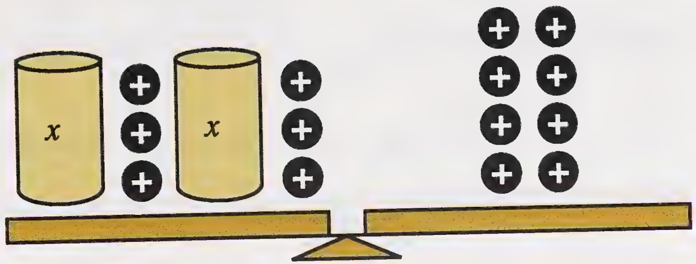
The scale is balanced.

So, $a = -3$.

Example 2

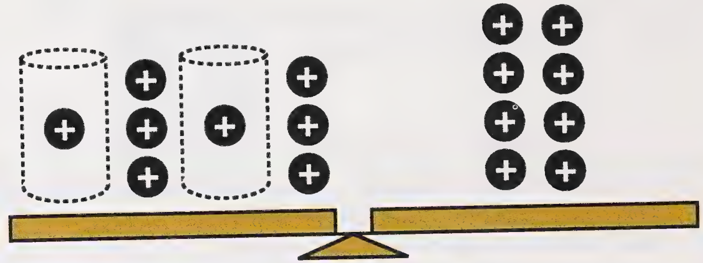
Solve $2(x + 3) = 8$.

Use a balance to model the equation.



Again you will have to guess, check, and revise before you discover that $x = 1$ is the solution.

To verify the solution, replace each x with $+1$.



Each side of the scale has a value of $+8$ if $x = +1$.

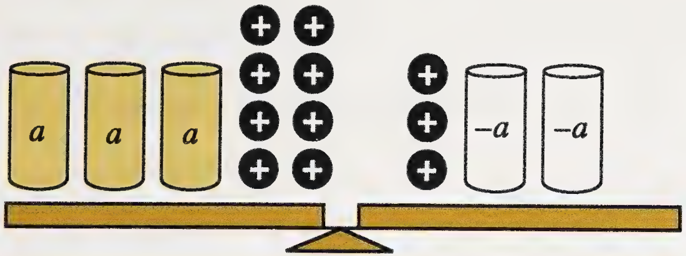
The scale is balanced.

So, $x = 1$.

Example 3

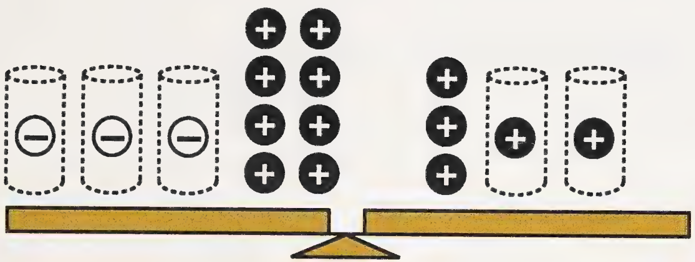
Solve $3a + 8 = 3 - 2a$.

Use a balance to model the equation.

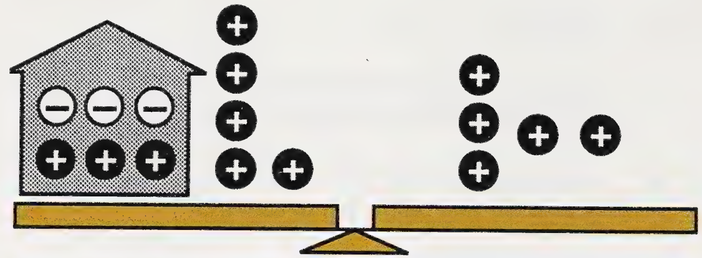


Use the guess-check-revise method to discover that the solution is $a = -1$.

To verify the solution, replace each a with -1 and each $-a$ with $+1$. Remember that a and $-a$ are opposites.



Simplify the equation by removing zeros.



The simplified scale looks like this.

Each side of the scale has a value of $+5$ if $a = -1$.

The scale is balanced.

So, $a = -1$.

Introductory Activities

Space for Your Work

1. a. Model $a + 4 = 5$.

   b. Solve the equation by inspection. (Find the value that makes the equation true.)

   c. Verify the solution. (Check to see if the equation is true for this value of the variable.)

2. a. Model $b - 2 = -5$.

   b. Solve the equation by inspection.

   c. Verify the solution.

3. a. Model $2c = -6$.

   b. Solve the equation by inspection.

   c. Verify the solution.

4. a. Model $3d = 9$.

   b. Solve the equation by inspection.

   c. Verify the solution.

5. a. Model $2m + 1 = 7$.

   b. Solve the equation by inspection.

   c. Verify the solution.

6. a. Model $3x - 1 = 2$.

   b. Solve the equation by inspection.

   c. Verify the solution.

7. a. Model $3x - 8 = 16$.

   b. Solve the equation by using the guess-check-revise method.

   c. Verify the solution.

8. a. Model $4x = 2x + 6$.

   b. Solve the equation by using the guess-check-revise method.

   c. Verify the solution.

Space for Your Work

9. a. Model $8 + 3x = 7x$.

   b. Solve the equation by using the guess-check-revise method.

   c. Verify the solution.

10. a. Model $4(n + 3) = 20$.

   b. Solve the equation by using the guess-check-revise method.

   c. Verify the solution.

11. a. Model $2x - 3 = 2 + 3x$.

   b. Solve the equation by using the guess-check-revise method.

   c. Verify the solution.

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

Working Together

In the Introductory Activities you used models, along with inspection and guess-check-revise methods, to solve equations. Now you will solve equations without using models.

Example 1

Solve the equation $y + 5 = 12$.

Solution

Think: 5 added to what number gives 12?

$$7 + 5 = 12$$

So, $y = 7$.

Check

LS	RS
$y + 5$	12
$= 7 + 5$	
$= 12$	

LS = RS

So, $y = 7$.

Example 2

Solve the equation $5k = 55$.

Solution

Think: 5 times what number is 55?

$$5 \times 11 = 55$$

So, $k = 11$.

Check

LS	RS
$5k$	55
$= 5 \times 11$	
$= 55$	

LS = RS

So, $k = 11$.

Practice Activities

Space for Your Work

Solve the following equations by using inspection or guess-check-revise methods. Do not use models.

1. $n + 8 = 12$
2. $p - 5 = -5$
3. $4b = -4$
4. $6 + x = 3$
5. $3w + 8 = 5$
6. $4t - 1 = 7$
7. $5x + 4 = 4x + 10$
8. $2(a - 7) = 4$

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

Solve the following equations by using inspection or guess-check-revise methods. Do not use models.

1. $\frac{a}{3} = 5$

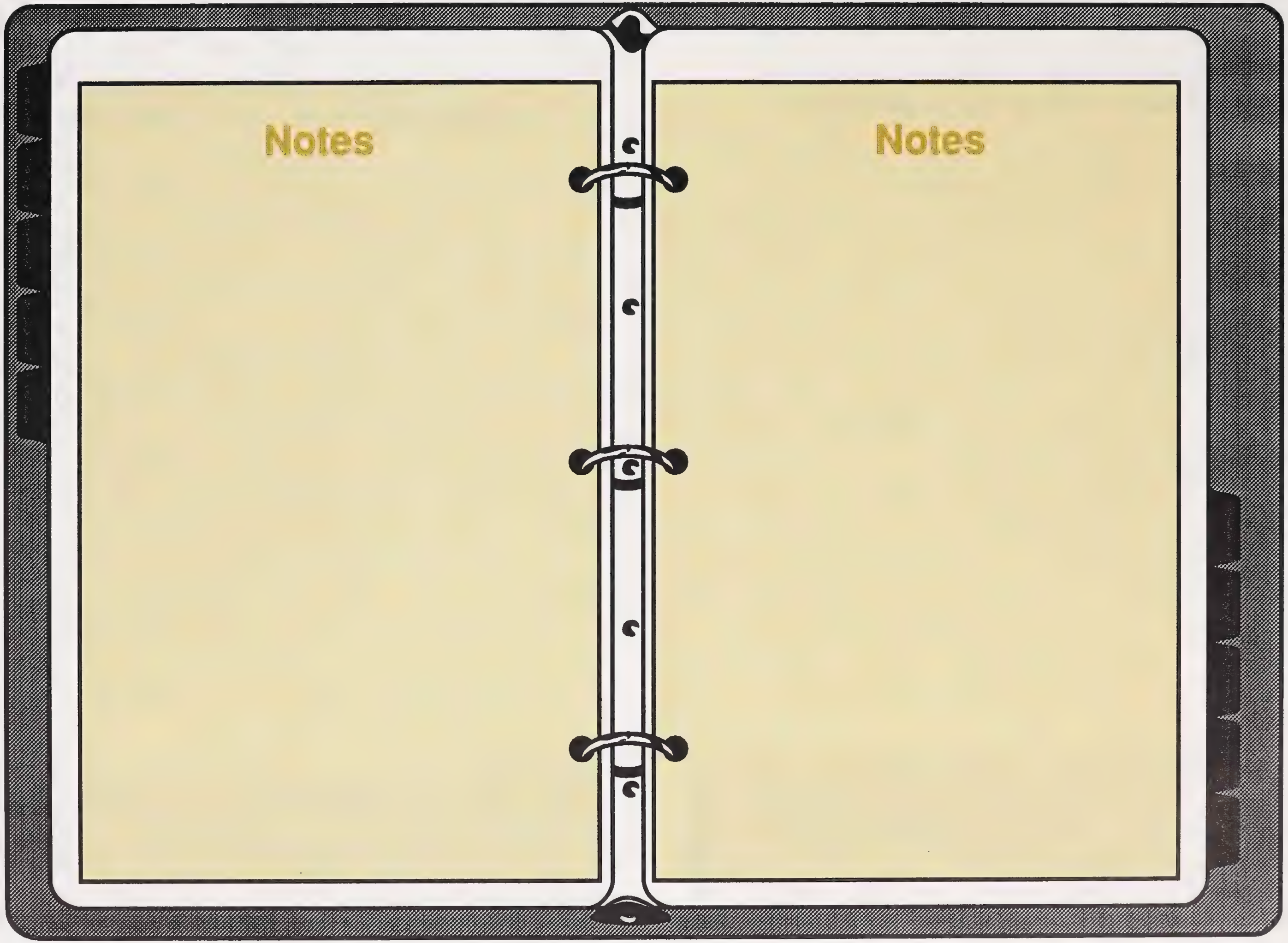
2. $\frac{b}{4} = 20$

3. $\frac{30}{c} = 6$

4. $\frac{r}{15} = \frac{1}{5}$

5. $\frac{10}{t} = \frac{2}{3}$

See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

SOLVING EQUATIONS USING ADDITIVE INVERSES

What Lies Ahead

In this section you will learn these skills.

- using learning aids and additive inverses to solve equations
- using a paper and pencil method and additive inverses to solve equations

In this section you will use these words.

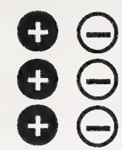
- additive inverse
- constant

Working Together

In the previous section you solved equations using inspection and guess-check-revise methods. In Sections 7 to 10 you will learn more systematic ways to solve equations.

In Module 2 you learned about additive inverses.

Because their sum is 0, + 3 and – 3 are **additive inverses**.



In this section you will use additive inverses to solve equations.

Video Activity

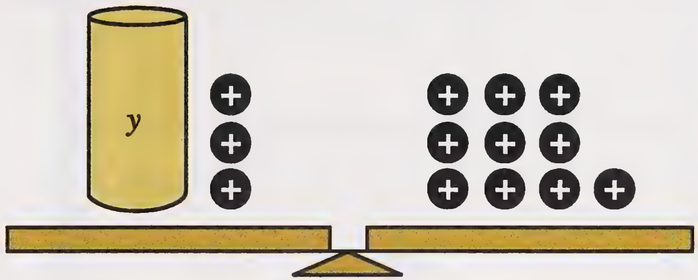
Watch the third segment of the video *MATH MOVES: Equations — Solving With One Step* and read the notes that follow. If you cannot view the video, simply read the notes that follow.

Example 1

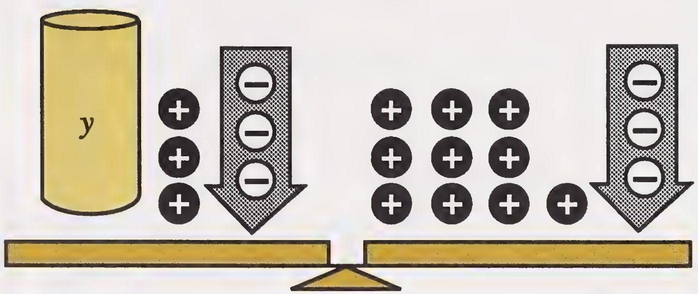
Solve the equation $y + 3 = 10$.

Solution

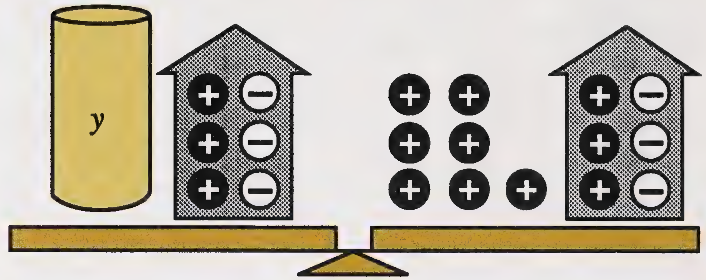
First model the equation.



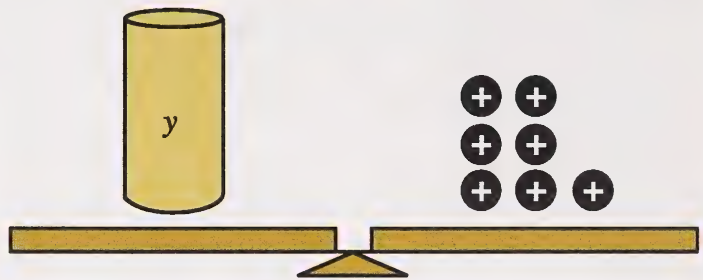
In order to isolate y, add -3 (the additive inverse of $+3$). To keep the balance, add -3 to both sides.



Simplify the equation by removing the zeros. This will not change the balance.

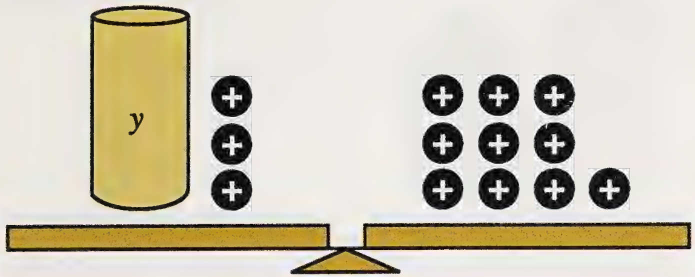


The result is this.

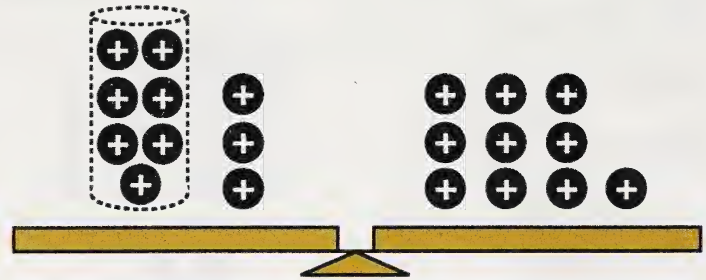


So, $y = 7$.

To verify the solution of $y + 3 = 10$, first model the equation.



Then replace y with $+7$.



Each side of the scale has a value of $+10$ if $y = +7$.

The scale is balanced.

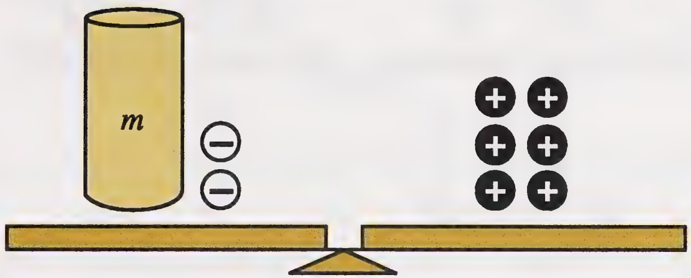
So, $y = 7$.

Example 2

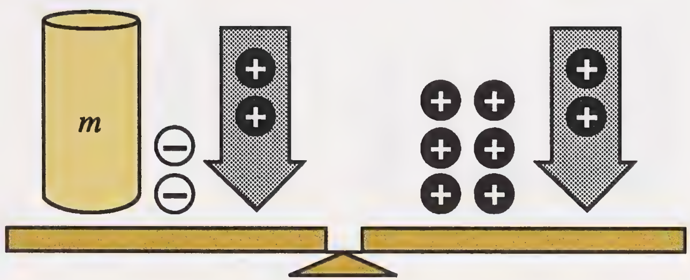
Solve the equation $m - 2 = 6$.

Solution

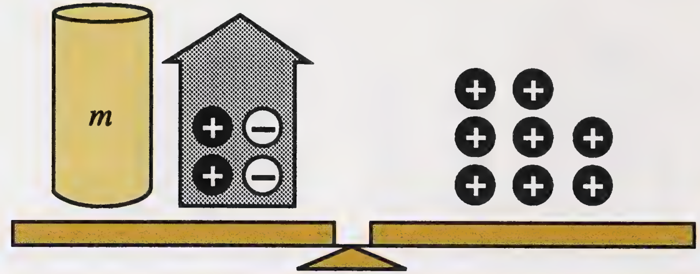
First model the equation.



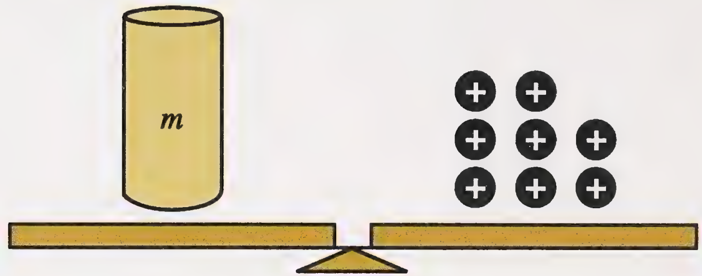
In order to isolate m, add $+2$ (the additive inverse of -2). To keep the balance, add $+2$ to both sides.



Simplify the equation by removing the zeros. This will not change the balance.



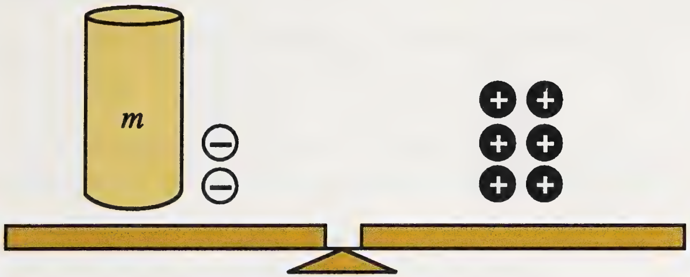
The result is this.



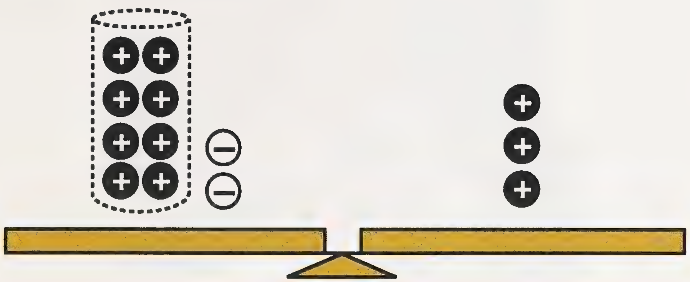
So, $m = 8$.

Verify the solution of $m - 2 = 6$.

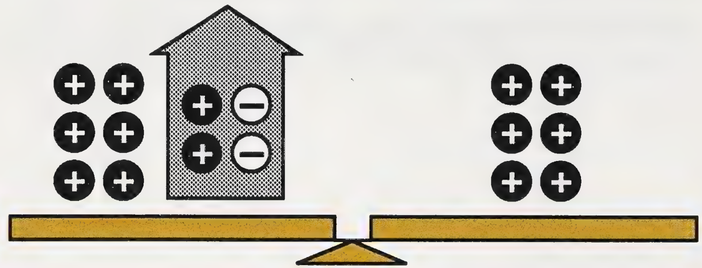
Model the equation.



Then replace m with 8.



Simplify by removing the zeros. This will not affect the balance.



The result is this.

Each side of the equation has a value of $+6$ if $m = 8$.

The scale is balanced.

So, $m = 8$.

Introductory Activities

Space for Your Work

Model these equations and solve the equations by isolating the variable. Be sure to verify your solutions.

1. $n + 7 = 9$

2. $b - 2 = 6$

3. $y - 1 = 1$

4. $q - 4 = -7$

5. $7 = m - 5$

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Solving Equations Using Paper and Pencil Methods

In the Introductory Activities you solved equations by isolating the variable. You used learning aids to do this.

It is not always convenient to rely on learning aids. So, you will now learn a paper and pencil method.

Study the following examples. The modelling method is shown on the left-hand side of the page. The paper and pencil method is shown on the right-hand side of the page. Notice the similarity between the two.

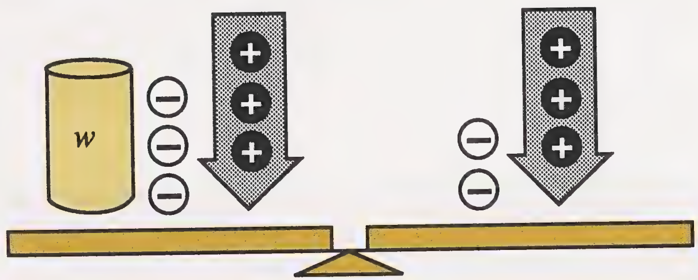
Example 1: Modelling Method

Solve $w - 3 = -2$ by isolating the variable.

Solution

First model the equation.

Next isolate the variable by adding + 3 (the additive inverse of – 3) to both sides.



Example 1: Paper and Pencil Method

Solve $w - 3 = -2$ by isolating the variable.

Solution

First write the equation.

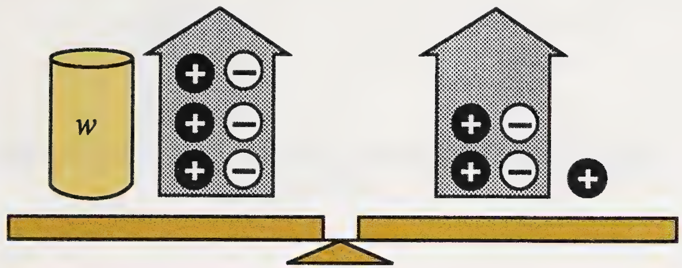
$$w - 3 = -2$$

Next isolate the variable by adding + 3 (the additive inverse of – 3) to both sides. You may use this vertical method of adding.

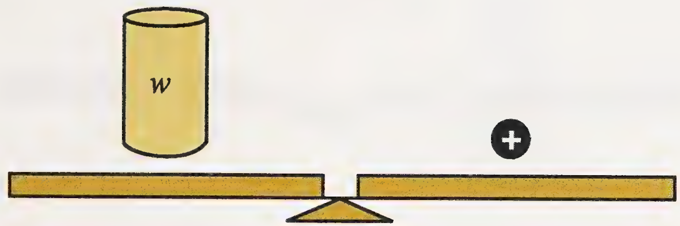
$$\begin{array}{rcl} w - 3 & = & -2 \\ +3 & = & +3 \\ \hline \end{array}$$

Example 1: Modelling Method Continued

Simplify both sides of the model by removing the zeros. This will not affect the balance.



So, this is the solution.



So, $w = +1$.

Example 1: Paper and Pencil Method Continued

Simplify the left-hand side of the equation by removing the zeros. This will not affect the balance.

$$\begin{array}{rcl} w \cancel{-3} & = & -2 \\ \cancel{+3} & = & +3 \\ \hline w & = & \end{array}$$

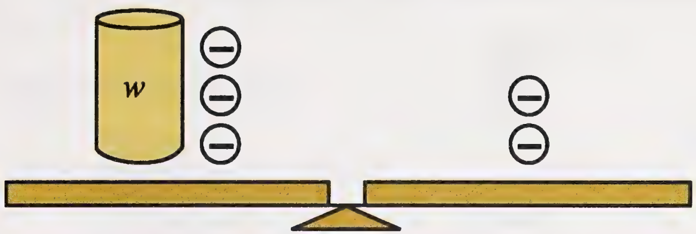
Simplify the right side of the equation to find the solution.

$$\begin{array}{rcl} w \cancel{-3} & = & -2 \\ \cancel{+3} & = & +3 \\ \hline w & = & +1 \end{array}$$

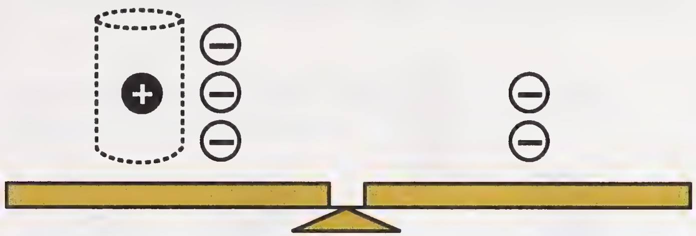
So, $w = +1$.

Verifying Example 1: Modelling Method

First model the equation.



Then replace w with + 1.



Verifying Example 1: Paper and Pencil Method

First write the sides of the equation in a chart.

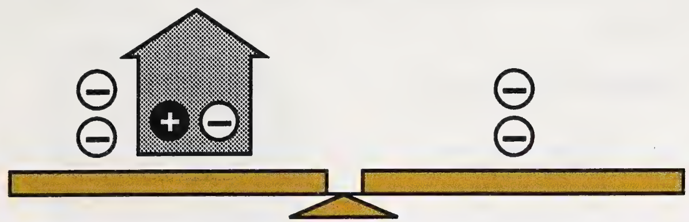
LS	RS
$w - 3$	-2
$= w + (-3)$	

Then replace w with + 1.

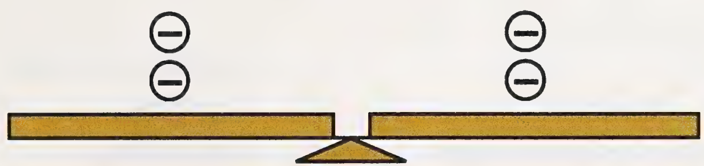
LS	RS
$w - 3$	-2
$= w + (-3)$	
$= 1 + (-3)$	

Verifying Example 1: Modelling Method Continued

Simplify the left-hand side by removing the zeros.



The left-hand side balances the right-hand side.



So, $w = 1$.

Verifying Example 1: Paper and Pencil Method Continued

Simplify the left-hand side of the equation.

LS	RS
$w - 3$	-2
$= w + (-3)$	
$= 1 + (-3)$	
$= -2$	

LS = RS

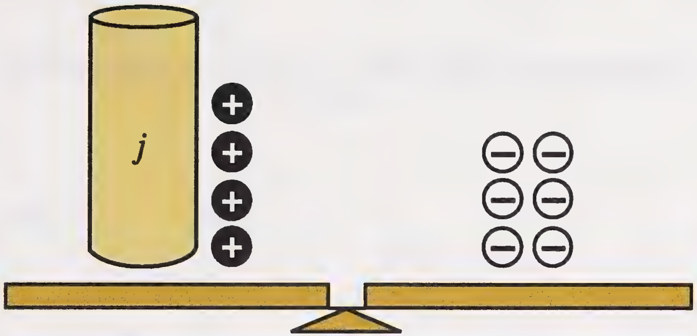
So, $w = 1$.

Example 2: Modelling Method

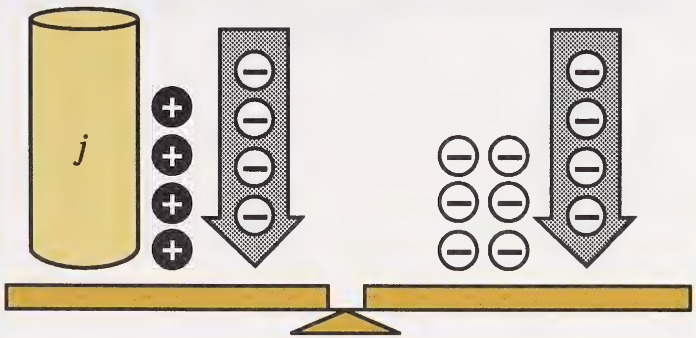
Solve the equation $j + 4 = -6$.

Solution

First model the equation.



Isolate the variable by adding – 4 (the additive inverse of + 4) to both sides.



Example 2: Paper and Pencil Method

Solve the equation $j + 4 = -6$.

Solution

First write the equation.

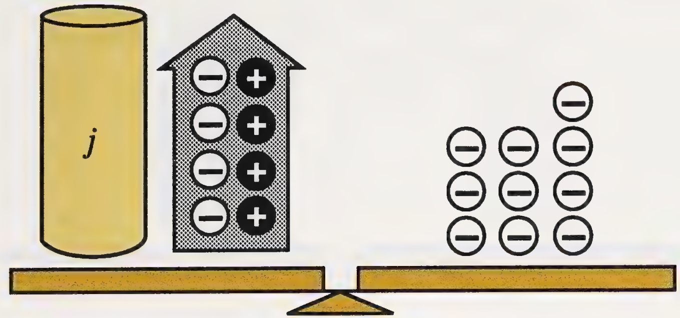
$$j + 4 = -6$$

Next isolate the variable by adding – 4 (the additive inverse of + 4) to both sides. You may use this horizontal method of adding.

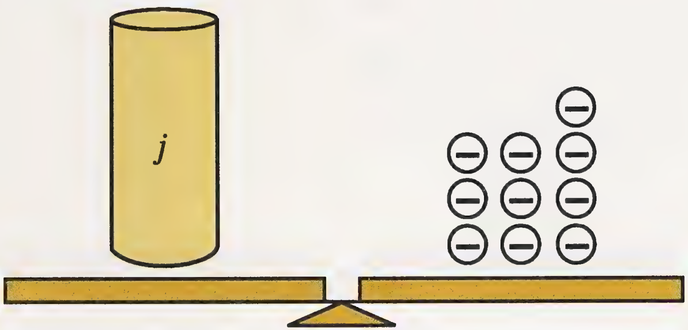
$$j + 4 - 4 = -6 - 4$$

Example 2: Modelling Method Continued

Then simplify the left-hand side of the model by removing the zeros. This will not affect the balance.



The solution is this.



So, $j = -10$.

Example 2: Paper and Pencil Method Continued

Simplify the left-hand side of the equation by removing the zeros. This will not affect the balance.

$$j \cancel{+4} \cancel{-4} = -6 - 4$$

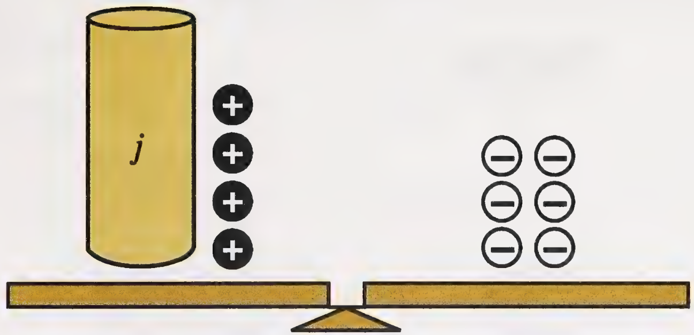
Simplify the right-hand side of the equation.

$$j \cancel{+4} \cancel{-4} = -6 - 4$$

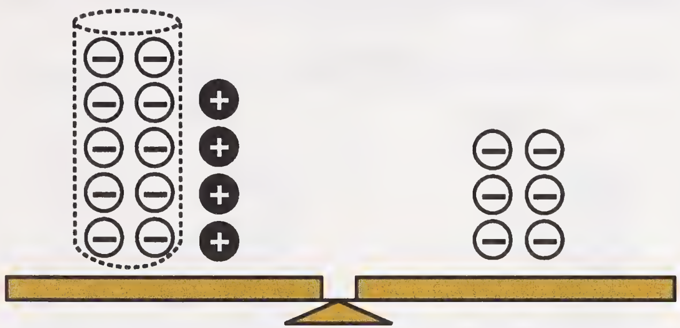
$$j = -10$$

Verifying Example 2: Modelling Method

First model the equation.



Then replace j with -10.



Verifying Example 2: Paper and Pencil Method

Write each side of the equation in a chart.

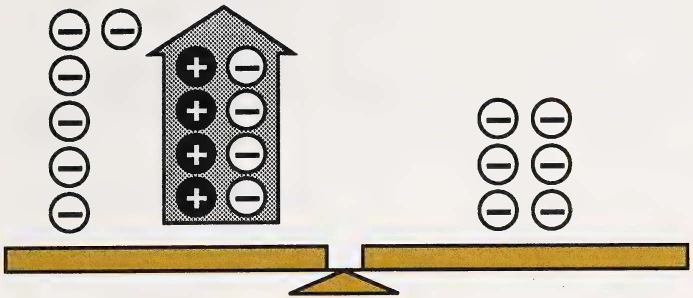
LS	RS
$j + 4$	-6

Then replace j with -10.

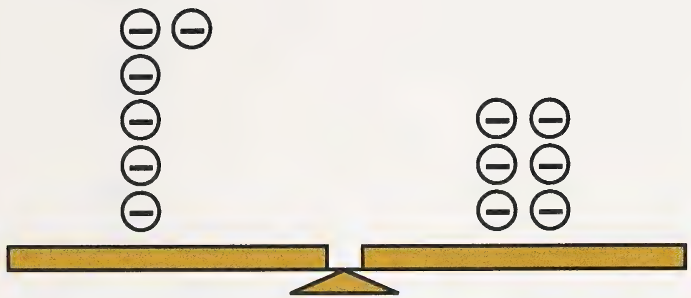
LS	RS
$j + 4$	-6
$= -10 + 4$	

Verifying Example 2: Modelling Method Continued

Simplify the left-hand side of the model by removing the zeros.



The result is this.



Both sides of the scale are balanced.

So, $j = -10$.

Verifying Example 2: Paper and Pencil Method Continued

Simplify the left-hand side of the equation.

LS	RS
$j + 4$	-6
$= -10 + 4$	
$= -6$	

LS = RS

So, $j = -10$.

Practice Activities

Space for Your Work

1. What numbers should be added to both sides to isolate the variable?

   a. $x + 2 = 7$

   b. $s + 4 = 9$

   c. $m + 9 = -13$

   d. $t - 5 = 7$

   e. $y - 2 = -8$

2. Solve the equations in Question 1 using paper and pencil methods. Verify your solutions.

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Solving Equations by Working Backwards

Another method used to solve equations is to work backwards by using a flow chart and an inverse flow chart.

Example 1: $n - 3 = 16$

Write the equation in flow chart form.

$$n \rightarrow \boxed{-3} \rightarrow 16$$

To solve the equation, use an inverse flow chart. That is, work backwards.

$$16 \rightarrow \boxed{+3} \rightarrow n$$

+ 3 is the opposite operation of – 3.

$16 + 3 = 19$

So, $n = 19$.

Verify your answer with a chart.

LS	RS
$n - 3$	16
$= 19 - 3$	
$= 16$	

LS = RS

The solution is indeed $n = 19$.

Example 2

Solve $b + 9 = -21$.

Write the equation in a flow chart.

$$b \rightarrow \boxed{+9} \rightarrow -21$$

To solve the equation, use an inverse flow chart.

$$-21 \rightarrow \boxed{-9} \rightarrow b$$

-9 is the opposite operation of $+9$.

$-21 - 9 = -30$

So, $b = -30$.

Verify your answer with a chart.

LS	RS
$b + 9$	-21
$= -30 + 9$	
$= -21$	

LS = RS

The solution is indeed $n = -30$.

Extra Practice

Space for Your Work

Use flow charts and inverse flow charts to solve these equations.

1. $s - 3 = 5$

2. $k + 12 = 39$

3. $2 + m = 8$

4. $s - 4 = 3$

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

Solve these equations using a paper and pencil method. Verify your solutions.

1. $x + 225 = 5$

2. $n - 8.5 = 12.3$

3. $q - \frac{1}{2} = \frac{3}{4}$

4. $p + 7.5 = 8.2$

5. $m - 5\frac{3}{4} = 3\frac{1}{4}$

6. $s - 383 = 117$

✓ See your learning facilitator to check your answers and to receive further instructions.

SOLVING EQUATIONS USING MULTIPLICATIVE INVERSES

What Lies Ahead

In this section you will learn these skills.

- using learning aids and multiplicative inverses to solve equations
- using paper and pencil methods and multiplicative inverses to solve equations

In this section you will use this word.

- multiplicative inverses

Working Together

You have learned to solve equations using additive inverses. In this section you will learn a method for solving equations using **multiplicative inverses**.

Video Activity

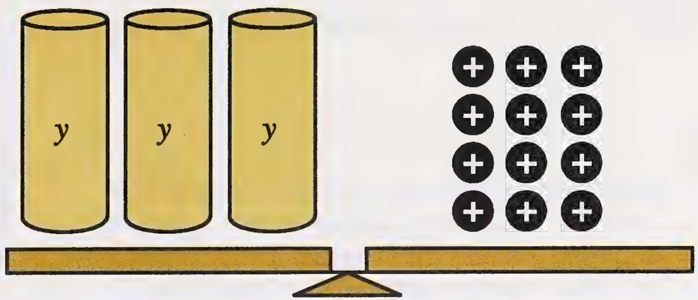
Watch the fourth segment of the video *MATH MOVES: Equations — Solving With One Step* and read the notes that follow. If you cannot view the video, simply read the notes that follow.

Example

Solve the equation $3y = 12$.

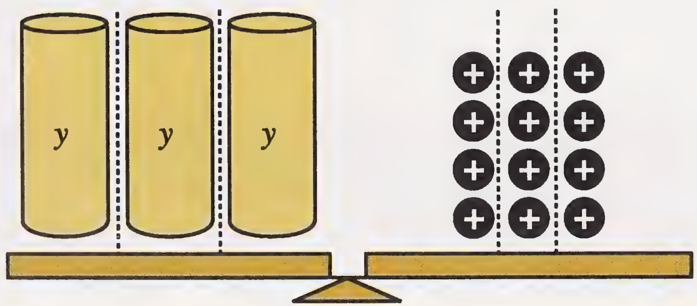
Solution

First model the equation.

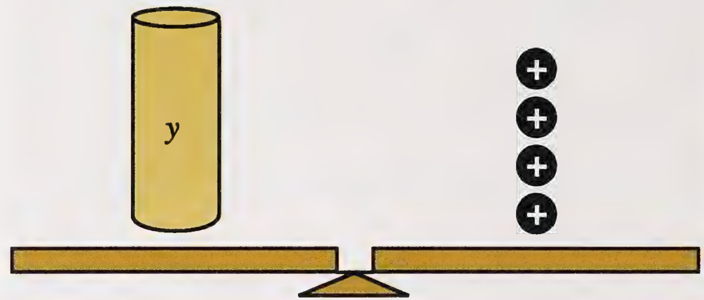


To isolate the variable, multiply each side by $\frac{1}{3}$ (the multiplicative inverse of 3).

Begin by dividing each side into three groups.

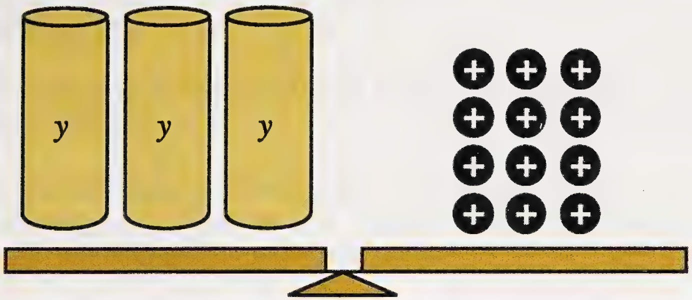


To solve the equation, examine only one of the three groups on each side.

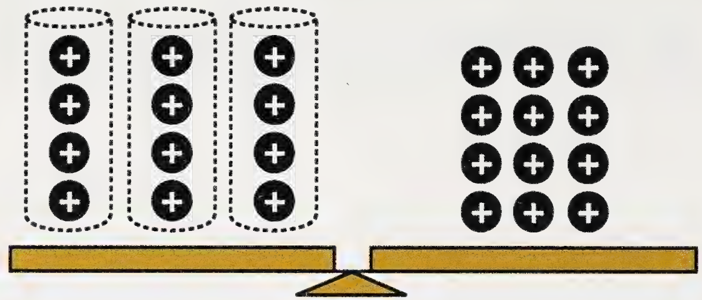


The solution is $y = 4$.

To verify the solution of $3y = 12$, you must first model the equation.



Then replace y with 4.



Each side of the equation has a value of $+12$ if $y = 4$.

The scale is balanced.

So, $y = 4$.

Introductory Activities

Space for Your Work

Model these equations and solve the equations by isolating the variable. Be sure to verify your solutions.

1. $2y = 6$

2. $2x = 6$

3. $2t = -2$

4. $3w = 12$

5. $10 = 5t$

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Solving Equations Using Paper and Pencil Methods

In the Introductory Activities you solved equations by isolating the variable through opposite operations. You used learning aids or models to do this.

It is not always convenient to rely on learning aids, so you will now learn a paper and pencil method to solve equations.

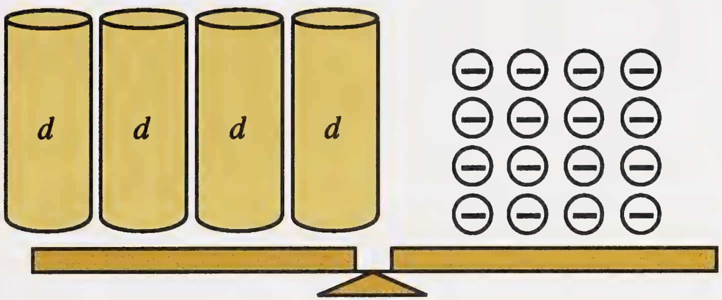
Study the following examples. On the left-hand side of the page is the modelling method. On the right-hand side of the page is the paper and pencil method. Notice the similarity between the two.

Example: Modelling Method

Solve the equation $4d = -16$.

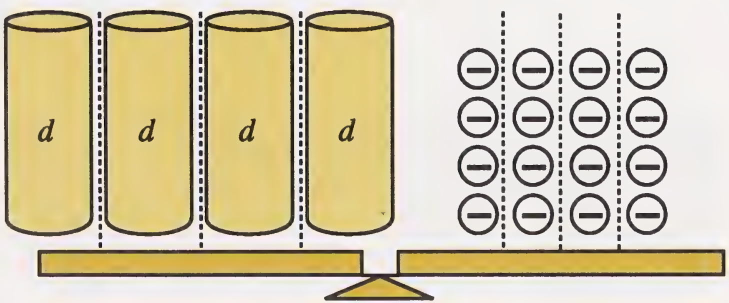
Solution

First model the equation.



To isolate the variable, multiply each side by $\frac{1}{4}$ (the multiplicative inverse of 4).

Begin by dividing each side into four groups.



Example: Paper and Pencil Method

Solve the equation $4d = -16$.

Solution

First write the equation.

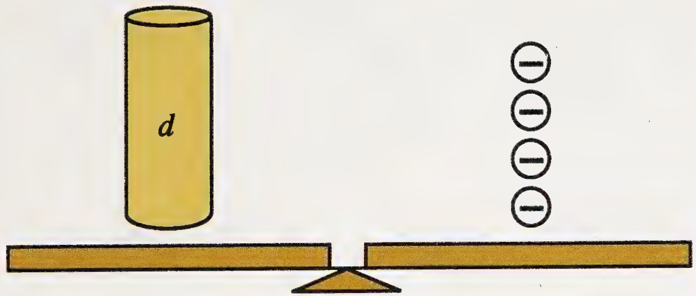
$$4d = -16$$

To isolate the variable, multiply both sides by $\frac{1}{4}$ (the multiplicative inverse of 4).

$$\frac{1}{4} \times 4d = \left(\frac{1}{4}\right) \times (-16)$$

Example: Modelling Method Continued

To solve the equation, examine only one of the three groups on each side.



The solution is $d = -4$.

Example: Paper and Pencil Method Continued

Simplify each side of the equation.

$$\frac{1}{\cancel{4}_{1}} \times \overset{1}{\cancel{4}}d = \left(\frac{1}{\cancel{4}_{1}}\right) \times (-\overset{-4}{\cancel{16}})$$

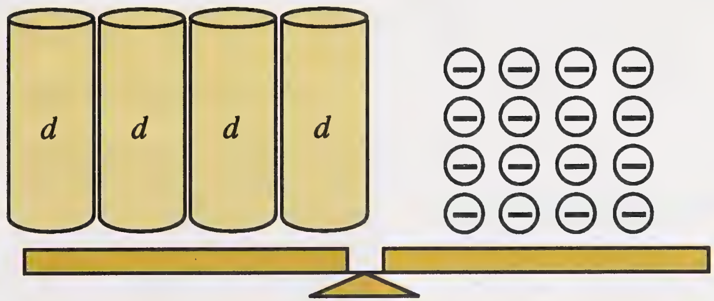
$$1d = -4$$

$$d = -4$$

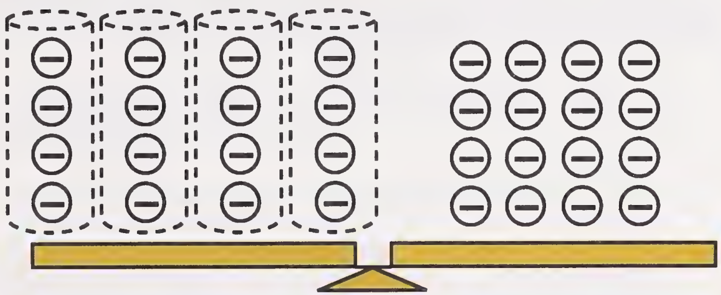
The solution is $d = -4$.

Verifying the Example: Modelling Method

First model the equation.



Then replace d with – 4.



The left-hand side equals the right-hand side.

So, the solution is $d = -4$.

Verifying the Example: Paper and Pencil Method

First write the sides of the equation in a chart.

LS	RS
	– 16

Then replace d with – 4 and simplify.

LS	RS
$4d$	– 16
$= 4 \times (-4)$	
$= -16$	

LS = RS

So, the solution is $d = -4$.

Practice Activities

Space for Your Work

1. What number should both sides be divided by to isolate the variable?

   a. $2y = 18$

   b. $4v = 32$

   c. $3m = -9$

   d. $-2f = -4$

   e. $5y = -10$

2. Solve the equations in Question 1 by using paper and pencil methods. Verify your solutions.

✓ See your learning facilitator to check your answers and to receive further instructions.

Working Together

Solving Equations by Working Backwards

You can solve equations like those in the Practice Activities by using flow charts and inverse flow charts.

Example

Solve the equation $4d = 144$.

Solution

Write the equation in a flow chart.

$$d \downarrow \boxed{\times 4} \downarrow 144$$

To solve the equation, use an inverse flow chart.

$$144 \downarrow \boxed{\div 4} \downarrow d$$

$\div$ 4 is the inverse operation of $\times$ 4.

$144 \div 4 = 36$

So, $d = 36$.

Use a chart to verify your answer.

LS	RS
$4d$	144
$= 4 \times 36$	
$= 144$	

LS = RS

So, the solution is $d = 36$.

Extra Practice

Space for Your Work

Use flow charts and inverse flow charts to solve these equations.

1. $9t = -72$

2. $3t = 30$

3. $44 = 4n$

4. $-10 = 2w$

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

Solve the equations using a paper and pencil method. Verify the solutions.

1. $4t = 6$

2. $3a = \frac{1}{2}$

3. $2r = \frac{3}{4}$

4. $2p = 14.4$

5. $3m = 0.9$

✓ See your learning facilitator to check your answers and to receive further instructions.

SOLVING MORE EQUATIONS USING MULTIPLICATIVE INVERSES

What Lies Ahead

In this section you will learn these skills.

- using learning aids and multiplicative inverses to solve more equations
- using paper and pencil methods and multiplicative inverses to solve more equations
- using clearing-denominator methods and cross-product methods to solve equations

Working Together

In Section 8 you solved equations in which the variable was multiplied by a whole number. Here is an example.

$$3a = 6$$

$3a$ means $3 \times a$.

In this section you will learn to solve equations in which the variable is multiplied by a fraction. Here are some examples.

$$\frac{a}{2} = 3$$

$\frac{a}{2}$ means $\frac{1}{2}a$ or $\frac{1}{2} \times a$.

$$\frac{x}{3} = \frac{1}{2}$$

$\frac{x}{3} = \frac{1}{3}x$ or $\frac{1}{3} \times x$.

You can solve some equations with fractions using multiplicative inverses.

Example: Modelling Method

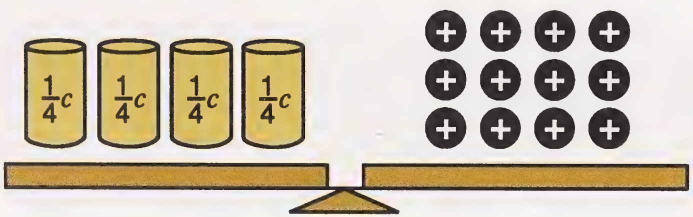
Solve $\frac{c}{4} = 3$ by isolating the variable.

$\frac{c}{4}$ means $\frac{1}{4}c$.

Solution

First model the equation.

The multiplicative inverse of $\frac{1}{4}$ is 4. So, to isolate c, multiply each side by 4.



Example: Paper and Pencil Method

Solve $\frac{c}{4} = 3$ by isolating the variable.

Solution

First write the equation.

$$\frac{c}{4} = 3$$

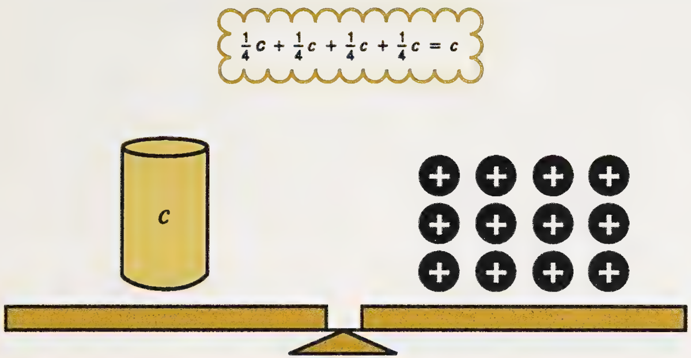
$\frac{c}{4}$ means $\frac{1}{4}c$.

The multiplicative inverse of $\frac{1}{4}$ is 4. So, to isolate the variable, multiply each side by 4.

$$\frac{c}{4} \times 4 = 3 \times 4$$

Example: Modelling Method Continued

Simplify the equation by combining like terms.



The solution is $c = 12$.

Example: Paper and Pencil Method Continued

Simplify the equation by performing the multiplication.

$$\frac{c}{\cancel{4}_{1}} \times \cancel{4}^{1} = 3 \times 4$$

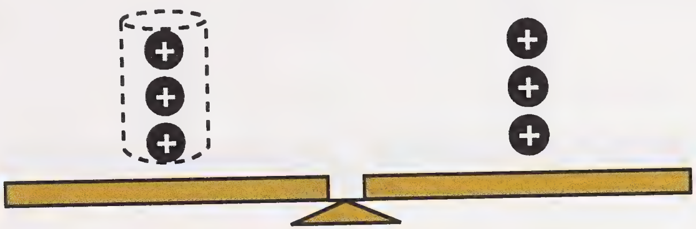
$$c = 12$$

The solution is $c = 12$.

Verifying the Example: Modelling Method

First model the equation.

Because $c = 12$, replace $\frac{1}{4}c$ with $\frac{1}{4} \times 12$ or 3.



Each side of the equation has a value of $+3$ if $c = 12$.

The scale is balanced.

So, $c = 12$.

Verifying the Example: Paper and Pencil Method

First write the sides of the equation in a chart.

LS	RS
$\frac{c}{4}$	3

Then replace c with 12 and simplify.

LS	RS
$\frac{c}{4}$	3
$= \frac{12}{4}$	
$= 3$	

LS = RS

So, the solution is $c = 12$.

Introductory Activities

Space for Your Work

1. Solve the following equations by using models. Be sure to verify each solution.

   a. $\frac{n}{5} = 6$

   b. $\frac{q}{11} = 3$

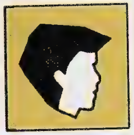
   c. $\frac{r}{2} = -12$

   d. $\frac{p}{8} = 2$

   e. $\frac{b}{7} = -1$

2. Solve the equations in Question 1 using paper and pencil methods.

> See your learning facilitator to check your answers and to receive further instructions.



Working Together

Study the following examples which have fractions on both sides of the equation.

Example

Solve $\frac{n}{4} = \frac{3}{2}$ by isolating the variable.

Solution

Write the equation.

$$\frac{n}{4} = \frac{3}{2}$$

$\frac{n}{4}$ means $\frac{1}{4}n$.

The multiplicative inverse of $\frac{1}{4}$ is 4. So, to isolate the variable, multiply each side by 4.

$$\frac{n}{4} \times 4 = \frac{3}{2} \times 4$$

Simplify the equation by performing the multiplication.

$$\frac{n}{\cancel{4}_1} \times \cancel{4}^1 = \frac{3}{\cancel{2}_1} \times \cancel{4}^2$$

$$n = 6$$

Verifying the Example

First write the equation in a chart.

LS	RS
$\frac{n}{4}$	$\frac{3}{2}$

Then replace n with 6 and simplify.

LS	RS
$\frac{n}{4}$	$\frac{3}{2}$
$= \frac{6}{4}$	
$= \frac{3}{2}$	

LS = RS

$$\frac{6}{4} \xrightarrow{\div 2} = \frac{3}{2}$$

So, the solution is $n = 6$.

Practice Activities

Space for Your Work

1. What number should both sides of the equation be multiplied by to isolate the variable?

   a. $\frac{n}{15} = \frac{2}{3}$

   b. $\frac{f}{9} = \frac{8}{3}$

   c. $\frac{y}{5} = \frac{30}{36}$

   d. $\frac{r}{7} = \frac{5}{8}$

   e. $\frac{3}{4} = \frac{c}{20}$

2. Solve the equations in Question 1. Verify your solutions.

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Solving Equations by Working Backwards

Flow charts and inverse flow charts are helpful in solving equations.

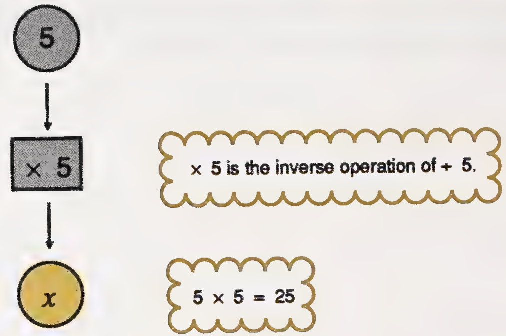
Example 1

Solve $\frac{x}{5} = 5$.

Solution

Write the equation in a flow chart.

To solve the equation, use an inverse flow chart.



So, $x = 25$.

Verify your answer by using a chart.

LS	RS
$\frac{x}{5}$	5
$= \frac{25}{5}$	
$= 5$	

LS = RS

So, the solution is indeed $x = 25$.

Extra Practice

Space for Your Work

Use flow charts and inverse flow charts to solve these equations.

1. $\frac{a}{3} = 2$

2. $\frac{b}{4} = 16$

3. $\frac{c}{2} = \frac{3}{4}$

4. $\frac{d}{5} = \frac{8}{25}$

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Here are two other ways to solve equations with fractions on both sides of the equation.

Example 1

Solve $\frac{p}{5} = \frac{1}{2}$.

Solution

Method 1

First write the equation.

$$\frac{p}{5} = \frac{1}{2}$$

Then multiply both sides of the equation by 10 (the product of the denominators 2 and 5). This is called **clearing the denominators**.

$$10 \times \frac{p}{5} = \frac{1}{2} \times 10$$

Simplify each side of the equation.

$$\overset{2}{\cancel{10}} \times \frac{p}{\underset{1}{\cancel{5}}} = \frac{1}{\underset{1}{\cancel{2}}} \times \overset{5}{\cancel{10}}$$

$$2p = 5$$

The multiplicative inverse of 2 is $\frac{1}{2}$. So, to isolate the variable, multiply both sides by $\frac{1}{2}$ and simplify.

$$2p = 5$$

$$\frac{1}{2} \times 2p = 5 \times \frac{1}{2}$$

$$\frac{1}{\underset{1}{\cancel{2}}} \times \overset{1}{\cancel{2}}p = 5 \times \frac{1}{2}$$

$$p = \frac{5}{2} \text{ or } 2.5$$

Method 2

First write the equation.

$$\frac{p}{5} = \frac{1}{2}$$

Then find **cross products**. This is a short cut to clearing the denominators.

$$\frac{p}{5} \times \frac{1}{2}$$

$$2 \times p = 5 \times 1$$

$$2p = 5$$

The multiplicative inverse of 2 is $\frac{1}{2}$. So, to isolate the variable, multiply both sides by $\frac{1}{2}$ and simplify.

$$\frac{1}{2} \times 2p = 5 \times \frac{1}{2}$$

$$\frac{1}{\cancel{2}_1} \times \cancel{2}^{1}p = 5 \times \frac{1}{2}$$

$$p = \frac{5}{2} \text{ or } 2.5$$

Verifying Example 1

First write the equation in a chart.

LS	RS
$\frac{p}{5}$	$\frac{1}{2}$

Then replace p with 2.5.

LS	RS
$\frac{p}{5}$	$\frac{1}{2}$
$= \frac{2.5}{5}$	$= 0.5$
$= 0.5$	

LS = RS

So, the solution is $p = 2.5$.

Example 2

Solve $\frac{3}{a} = \frac{1}{4}$.

Solution

Method 1

First write the equation.

$$\frac{3}{a} = \frac{1}{4}$$

The product of the denominators is $4a$. So, multiply both sides of the equation by $4a$ and simplify.

$$4a \times \frac{3}{a} = \frac{1}{4} \times 4a$$

$$\overset{1}{4\cancel{a}} \times \frac{3}{\underset{1}{\cancel{a}}} = \frac{1}{\underset{1}{\cancel{4}}} \times \overset{1}{\cancel{4}}a$$

$$12 = a$$

Method 2

First write the equation.

$$\frac{3}{a} = \frac{1}{4}$$

Then find the cross products. This is a short cut to clearing the denominators.

$$\frac{3}{a} \times \frac{1}{4}$$

$$1 \times a = 3 \times 4$$

$$a = 12$$

You can verify Example 2 by using a chart.

LS	RS
$\frac{3}{a}$	$\frac{1}{4}$
$= \frac{3}{12}$	
$= \frac{1}{4}$	

$$\frac{3}{12} = \frac{1}{4}$$

+3

+3

LS = RS

So, the solution is $a = 12$.

Concluding Activities

Space for Your Work

Solve the following equations by clearing the denominators or by using cross products. Be sure to verify your solutions.

1. $\frac{h}{7} = \frac{5}{2}$

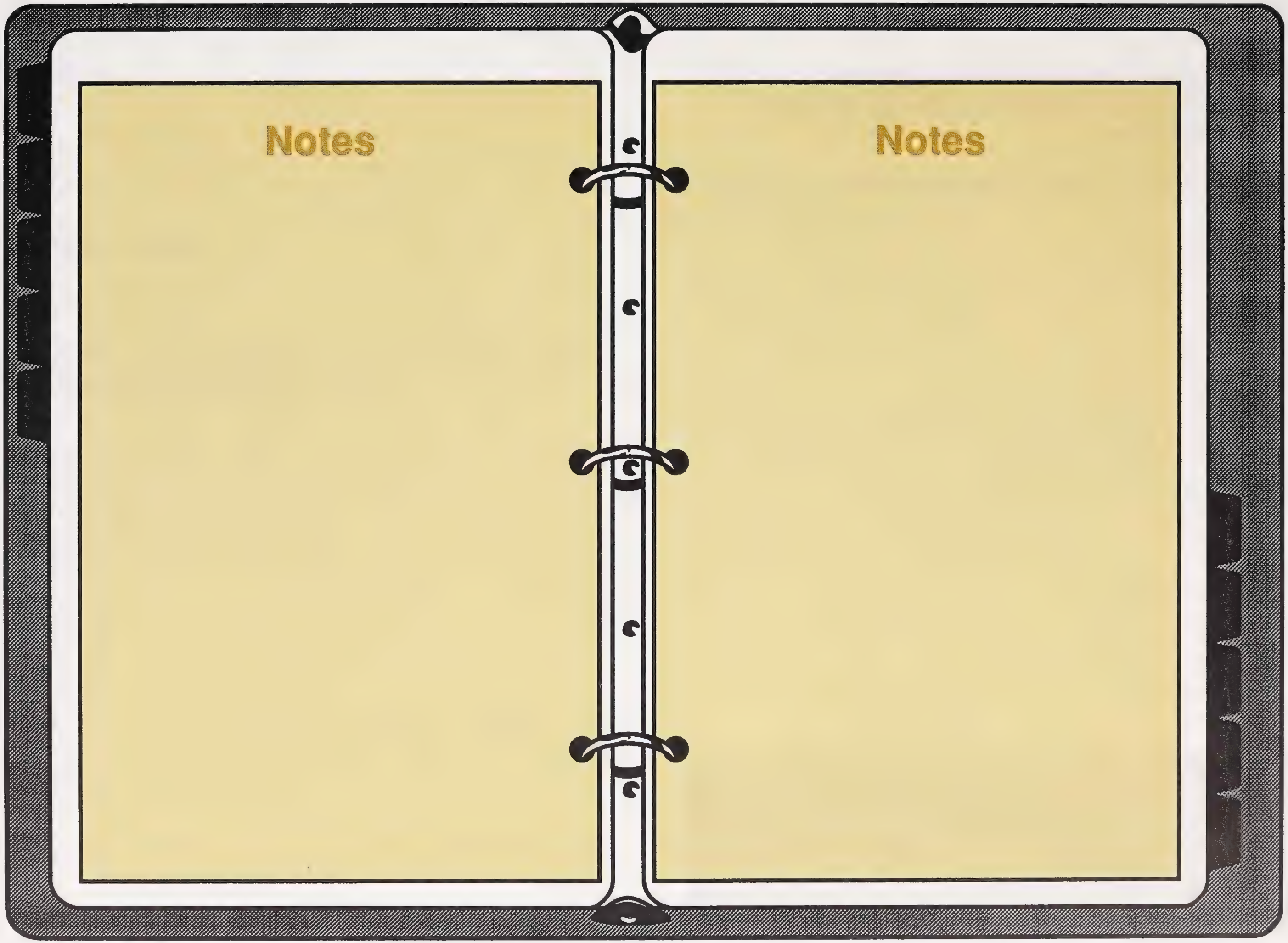
2. $\frac{5}{t} = \frac{9}{8}$

3. $\frac{n}{2} = \frac{1}{4}$

4. $\frac{4}{m} = \frac{1}{3}$

5. $\frac{w}{6} = \frac{1}{3}$

See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

SOLVING EQUATIONS USING ADDITIVE AND MULTIPLICATIVE INVERSES

What Lies Ahead

In this section you will learn these skills.

- solving more complex equations using learning aids
- solving more complex equations using a procedure with paper and pencil

Working Together

In Section 7 you solved equations using additive inverses. In Section 8 you solved equations using multiplicative inverses.

In this section you will solve equations using both additive inverses and multiplicative inverses.

Video Activity

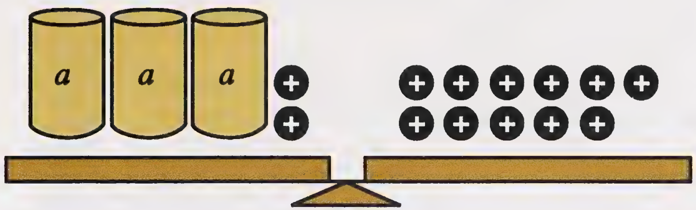
Watch the first segment of the video *MATH MOVES: Equations — Solving With More Than One Step* and read the notes that follow. If you cannot watch the video, simply read the following notes.

Example 1

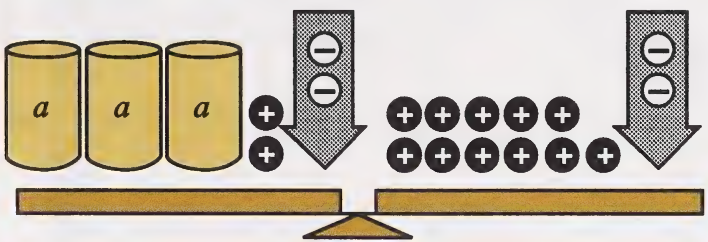
Solve the equation $3a + 2 = 11$.

Solution

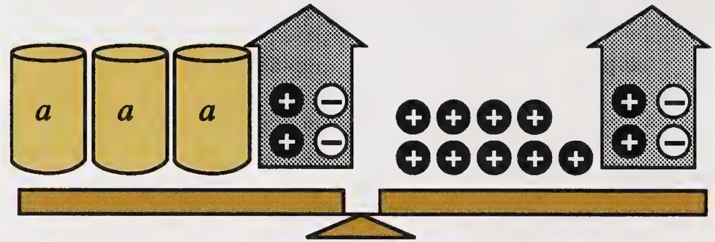
First model the equation.



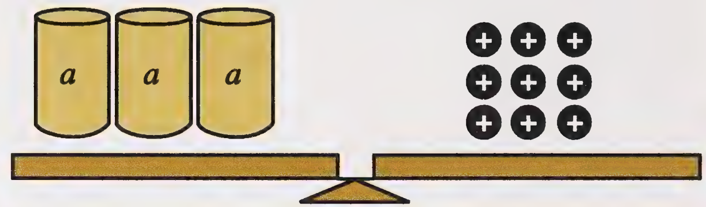
In order to isolate a, add -2 (the additive inverse of $+2$) to each side.



Simplify the equation by removing the zeros. This will not change the balance.

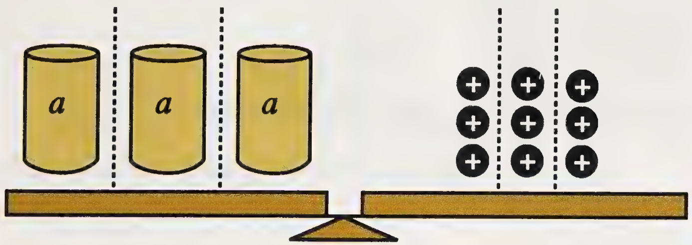


The result is this.



Example 1 Continued

In order to isolate a, divide each side into three groups.

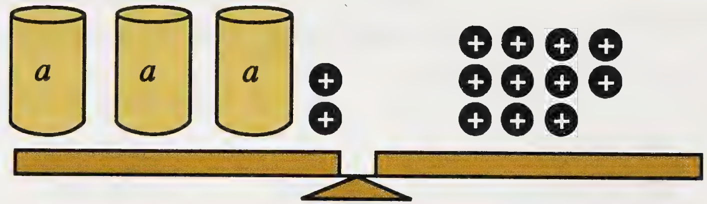


To solve the equation, examine only one of the groups on each side.

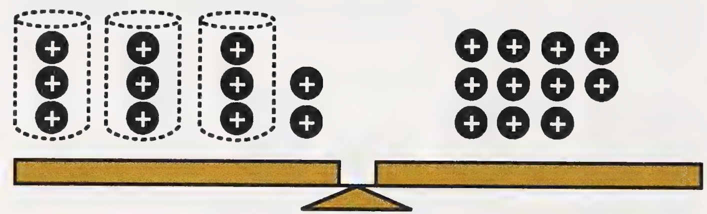
The solution is $a = +3$.

You can verify Example 1 to check your answer.

First model the equation.



Then replace a with 3.



Both sides have a value of $+11$ if $a = 3$.

The scale is balanced.

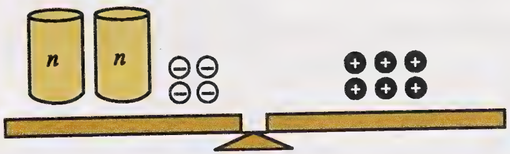
The solution is $a = 3$.

Example 2

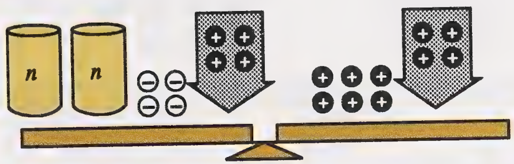
Solve $2n - 4 = 6$.

Solution

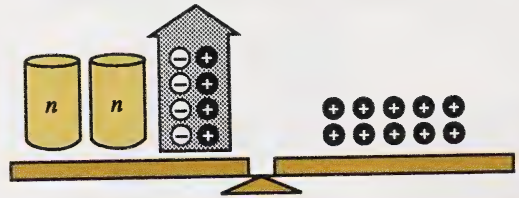
First model the equation.



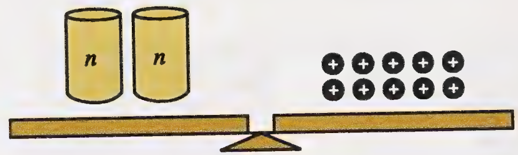
To isolate n, add $+ 4$ (the additive inverse of $- 4$) to each side.



Simplify the equation by removing the zeros. This will not change the balance.

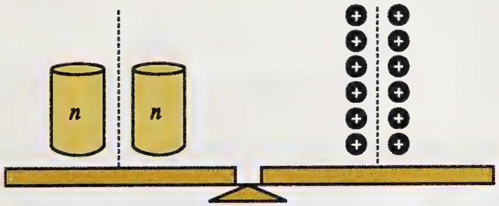


The result is this.

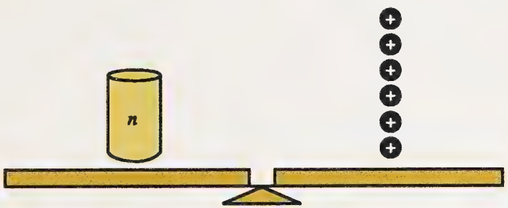


Example 2 Continued

In order to isolate n, divide each side into two groups.

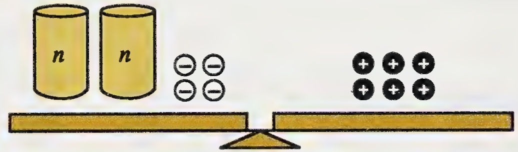


To solve the equation, examine only one group on each side.

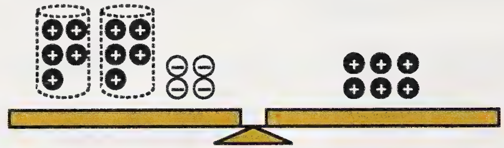


The solution is $n = 5$.

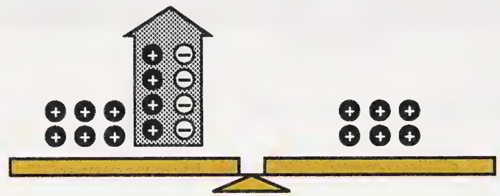
To verify Example 2, first model the equation.



Then replace n with 5.



Simplify by removing the zeros.



This is the result.

Both sides have a value of +6 if $n = 5$.

The solution is $n = 5$.

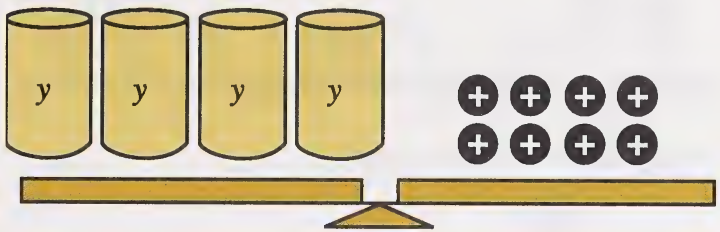
Some equations require you to collect like terms. Look at the following example.

Example

Solve the equation $2y + 2y = 8$.

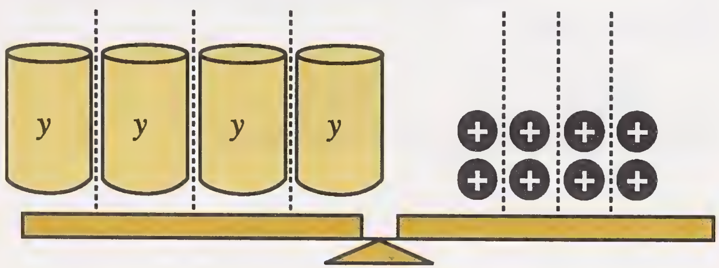
Solution

First model the equation.



The model shows that $2y + 2y = 4y$.

In order to isolate y, divide each side into four groups.

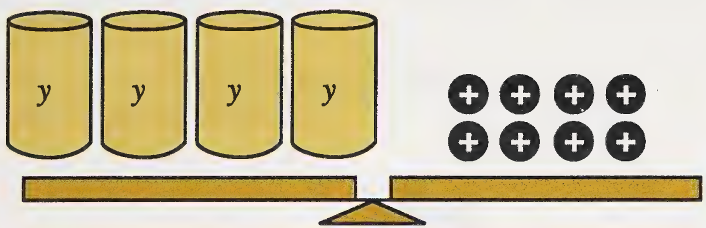


To solve the equation, examine only one of the groups on each side.

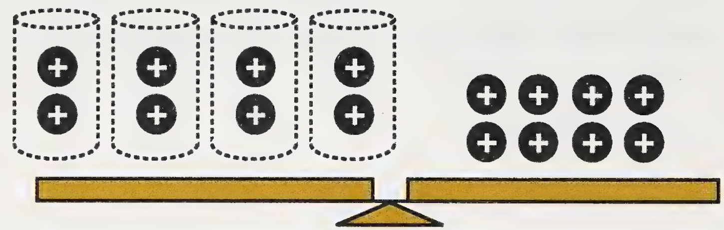
The solution is $y = 2$.

Verifying the Example

To verify the solution of $2y + 2y = 8$, first model the equation.



Then replace y with 2.



Both sides have a value of +8 if $y = 2$.

The scale is balanced.

The solution is $y = 2$.

Introductory Activities

Space for Your Work

Model these equations by isolating the variable. Be sure to verify your solutions.

1. $3q - 3 = 6$

2. $3y + 4 = 13$

3. $2b + 6 = 12$

4. $p + 3p = 8$

5. $5k - 2k = 6$

See your learning facilitator to check your answers and to receive further instructions.

Working Together

Solving Equations by Using Paper and Pencil Methods

You will now learn to solve the equations by using a paper and pencil method.

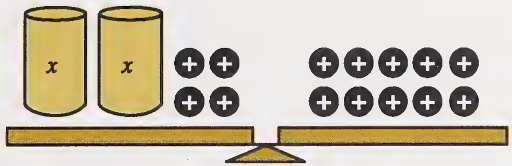
Study the following examples. On the left-hand side of the page is the modelling method. On the right-hand side of the page is the paper and pencil method. Notice the similarity between the two methods.

Example 1: Modelling Method

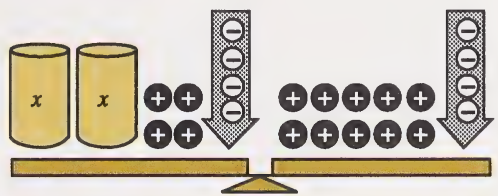
Solve the equation $2x + 4 = 10$.

Solution

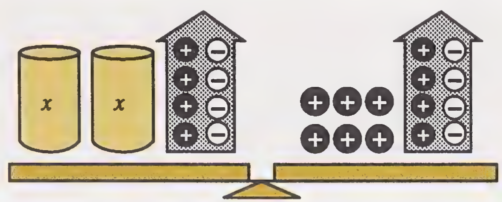
First model the equation.



To isolate x, add – 4 (the additive inverse of 4) to each side.



Simplify the equation by removing the zeros.



Example 1: Paper and Pencil Method

Solve the equation $2x + 4 = 10$.

Solution

First write the equation.

$$2x + 4 = 10$$

Add – 4 (the additive inverse of 4) to both sides of the equation.

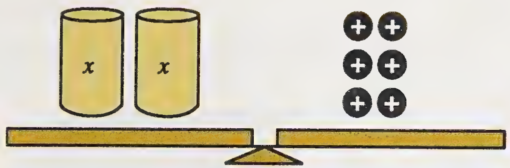
$$2x + 4 + (-4) = 10 + (-4)$$

Simplify the left-hand side of the equation by removing the zeros.

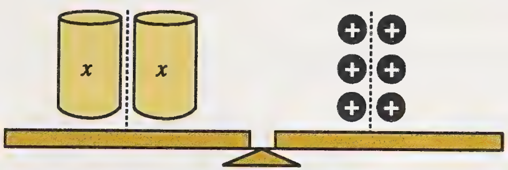
$$2x \cancel{+4} + \cancel{(-4)} = 10 + (-4)$$

Example 1: Modelling Method Continued

This is the result.



To isolate x, divide each side into two groups.



To solve the equation, examine only one group on each side.

The solution is $x = 3$.

Example 1: Paper and Pencil Method Continued

Simplify the right-hand side of the equation. This is the result.

$$2x = 6$$

The multiplicative inverse of 2 is $\frac{1}{2}$. So, to isolate x, multiply both sides by $\frac{1}{2}$.

$$\frac{1}{2} \times 2x = \frac{1}{2} \times 6$$

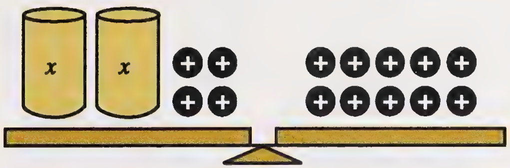
Simplify both sides of the equation.

$$\frac{1}{\cancel{2}_{1}} \times \overset{1}{\cancel{2}}x = \overset{3}{\cancel{6}} \times \frac{1}{\cancel{2}_{1}}$$

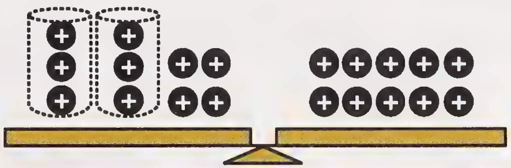
$$x = 3$$

Verifying Example 1: Modelling Method

First model the equation.



Then replace x with 3.



Both sides of the equation have a value of + 10 if $x = 3$.

The scale is balanced.

The solution is $x = 3$.

Verifying Example 1: Paper and Pencil Method

Write the equation in a chart.

LS	RS
$2x + 4$	10

LS	RS
$2x + 4$	10
$= 2 \times 3 + 4$	
$= 6 + 4$	
$= 10$	

LS = RS

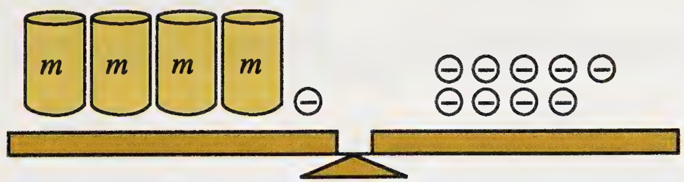
So, the solution is indeed $x = 10$.

Example 2: Modelling Method

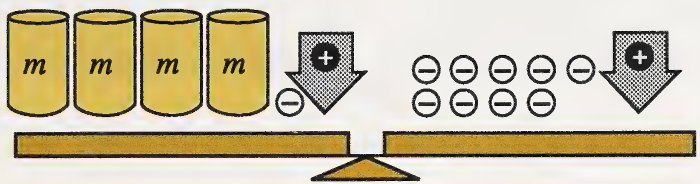
Solve the equation $4m - 1 = -9$ by isolating m.

Solution

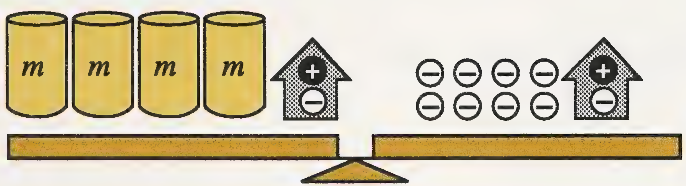
First model the equation.



Add + 1 (the additive inverse of – 1) to both sides.



Simplify the equation by removing the zeros.



Example 2: Paper and Pencil Method

Solve the equation $4m - 1 = -9$.

Solution

First write the equation.

$$4m - 1 = -9$$

Add + 1 (the additive inverse of – 1) to both sides of the equation.

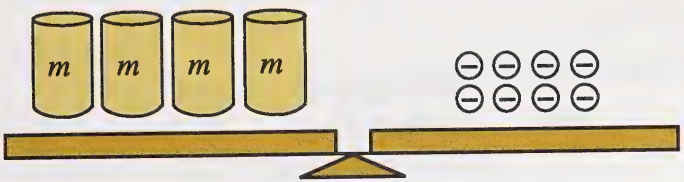
$$4m - 1 + 1 = -9 + 1$$

Simplify the equation by removing the zeros.

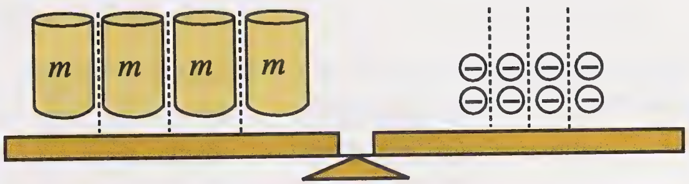
$$4m - \cancel{1} + \cancel{1} = -9 + 1$$

Example 2: Modelling Method Continued

This is the result.



To isolate m, divide each side into four groups.



To solve the equation, examine only one of the groups on each side.

The solution is $m = -2$.

Example 2: Paper and Pencil Method Continued

This is the result.

$$4m = -8$$

The multiplicative inverse of 4 is $\frac{1}{4}$. So, to isolate m, multiply each side by $\frac{1}{4}$.

$$\frac{1}{4} \times 4m = \frac{1}{4} \times (-8)$$

Simplify both sides of the equation.

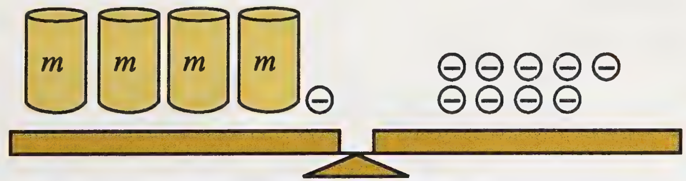
$$\frac{1}{\cancel{4}_{1}} \times \cancel{4}^{1} m = \frac{1}{\cancel{4}_{1}} \times (-\cancel{8}^{-2})$$

$$m = -2$$

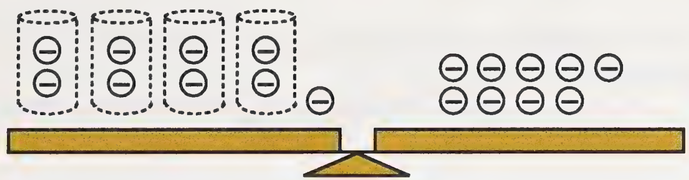
The solution is $m = -2$.

Verifying Example 2: Modelling Method

First model the equation $4m - 1 = -9$.



Then replace m with -2.



Both sides have a value of -9 if $m = -2$.

The scale is balanced.

So, the solution is $m = -2$.

Verifying Example 2: Paper and Pencil Method

First write the equation.

$$4m - 1 = -9$$

$$\text{or } 4m + (-1) = -9$$

Then replace m with -2.

LS	RS
$4m - 1$	-9
$= 4m + (-1)$	
$= 4 \times (-2) + (-1)$	
$= -8 + -1$	
$= -9$	

LS = RS

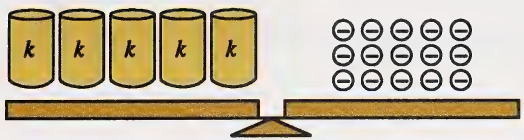
So, the solution is indeed $m = -2$.

Example 3: Modelling Method

Solve $3k + 2k = -15$ by isolating k.

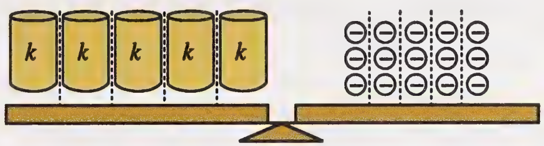
Solution

First model the equation.



The model shows $3k + 2k = +5k$.

Next isolate the variable by dividing each side into five groups.



To solve the equation, examine only one of the groups on each side.

The solution is $k = -3$.

Example 3: Paper and Pencil Method

Solve $3k + 2k = -15$ by isolating k.

Solution

First write the equation.

$$3k + 2k = -15$$

Then collect the like terms.

$$5k = -15$$

The multiplicative inverse of 5 is $\frac{1}{5}$. So, to isolate k, multiply both sides by $\frac{1}{5}$.

$$\frac{1}{5} \times 5k = \frac{1}{5} \times (-15)$$

Simplify each side of the equation.

$$\frac{1}{\cancel{5}_1} \times \cancel{5}^{1} k = \frac{1}{\cancel{5}_1} \times (-\cancel{15}^{3})$$

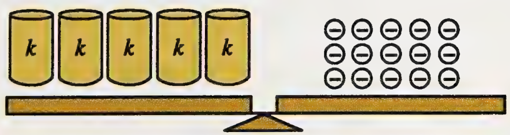
$$k = -3$$

The solution is $k = -3$.

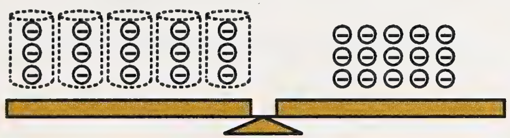
Verifying Example 3: Modelling Method

Verify the solution to $3k + 2k = -15$.

First model the equation.



Then replace k with -3.



Both sides have a value of -15 if $k = -3$.

The scale is balanced.

So, the solution is $k = -3$.

Verifying Example 3: Paper and Pencil Method

Verify the solution to $3k + 2k = -15$.

First write the equation.

$$3k + 2k = -15$$

Then replace k with -3.

LS	RS
$3k + 2k$	-15
$= 3 \times (-3) + 2 \times (-3)$	
$= -9 + -6$	
$= -15$	

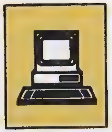
LS = RS

So, the solution is indeed $k = -3$.

Practice Activities

Space for Your Work

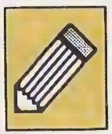
Computer Alternative



1. Do Lesson 8 and Lesson 10 on the disk *Pre-Algebra* from the package *Computer Drill and Instruction: Mathematics, Level D* (SRA).

Print Alternative

2. Solve the equations by using a paper and pencil method. Be sure to verify your solutions.



   a. $5x + 6 = 31$

   b. $6x - 3 = 15$

   c. $8a + 6 = 22$

   d. $3c - 3 = -24$

   e. $11n + 44 = 0$

3. Solve the following conditions. Place the letter that goes with each condition below the solution in the boxes below.

A $8y - 5y = 30$

L $4a + 2a + 3a = 18$

V $3x + 5x - 2x = 36$

T $9b - 4b + 2b = 77$

E $10c + 4c - 6c = 64$

H $4m + 3m = 84$

O $8x + 2x - 2 = 28$

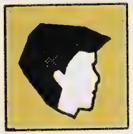
I $12y - 11y = 8 - 3$

M $-10a + 5a + 7 - 5 = -33$

5	4	2	3	6	8	9	7	10	11	12

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.



Working Together

Solving Equations by Working Backwards

Another method to solve equations is to use flow charts and inverse flow charts. Flow charts include all the steps that are normally used for solving equations but you do not have to write out all the steps.

Example

Solve $2n - 8 = 18$.

Write the equation in flow chart form.

$$n \rightarrow \boxed{\times 2} \rightarrow \boxed{-8} \rightarrow 18$$

To solve the equation, use an inverse flow chart. Work backwards using inverse operations. Do the operations in the order in which they appear on the inverse flow chart.

$$18 \rightarrow \boxed{+8} \rightarrow \boxed{\div 2} \rightarrow n$$

+ 8 is the opposite operation of − 8.

÷ 2 is the opposite operation of × 2.

$18 + 8 = 26$
$26 \div 2 = 13$

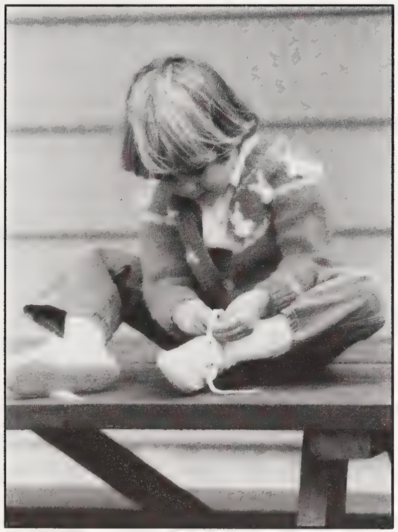
So, $n = 13$.

Notice that the flow chart uses the rules for order of operations — BEDMAS. In the example, multiplication is done before subtraction.

The inverse flow chart uses the opposite order. In the example, addition is done before division.

The inverse flow chart undoes what was done in the flow chart.

This doing and undoing process is similar to getting dressed and undressed. You put on your socks and then your shoes when you get dressed. You take off your shoes and then your socks when you undress.



WESTFILE INC.

Verifying the Example

First write the equation in a chart.

LS	RS
$2n - 8$	18

Then replace n with 13.

LS	RS
$2n - 8$	18
$= 2 \times 13 - 8$	
$= 26 - 8$	
$= 18$	

LS = RS

The solution is $n = 13$.

Extra Practice

Space for Your Work

Solve the following equations using flow charts and inverse flow charts. Use your calculator to help you if you wish.

1. $3a - 2 = 7$

2. $2b + 1 = -9$

3. $3c - 4 = 5$

4. $4b - 1 = -9$

5. $2d + 1 = 5$

See your learning facilitator to check your answers and to receive further instructions.

Concluding Activities

Space for Your Work

Solve the following equations by using additive and multiplicative inverses. Verify the solutions.

1. $2a + \frac{1}{2} = \frac{3}{4}$

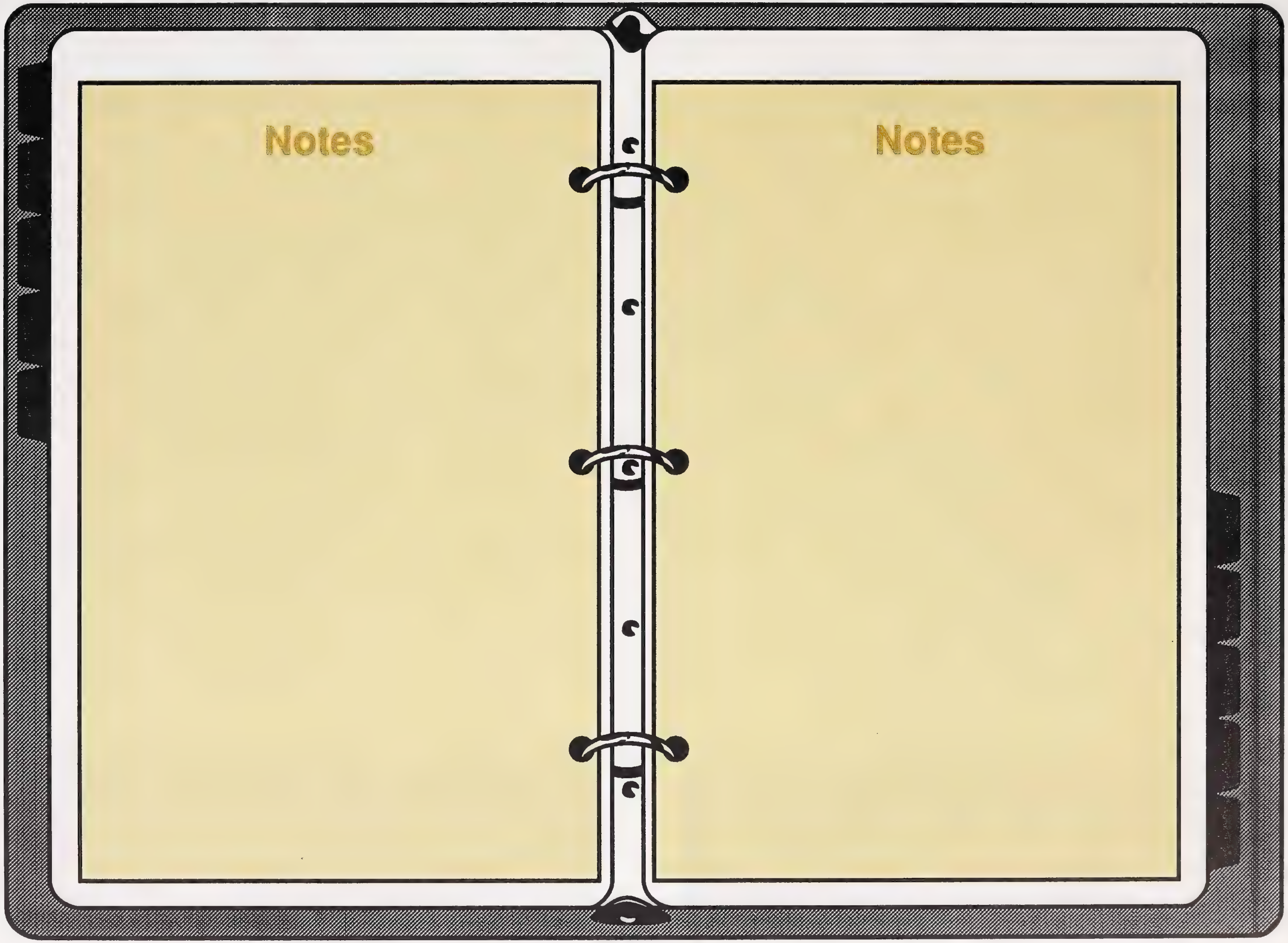
2. $3b - 1 = 8.3$

3. $5c + 1.5 = -10$

4. $2.5 = 1 + 3d$

5. $t + 1 + 3t = 9$

See your learning facilitator to check your answers and to receive further instructions.


Notes
Notes

EQUATIONS WITH TWO VARIABLES

What Lies Ahead

In this section you will learn these skills.

- modelling equations with two variables
- making a table of values for equations with two variables
- graphing equations

In this section you will use these terms.

- horizontal axis
- vertical axis
- coordinates
- coordinate plane
- ordered pairs
- plot
- origin

Working Together

So far in this module you have worked with equations which have only one variable. In this section you will work with equations with two variables.

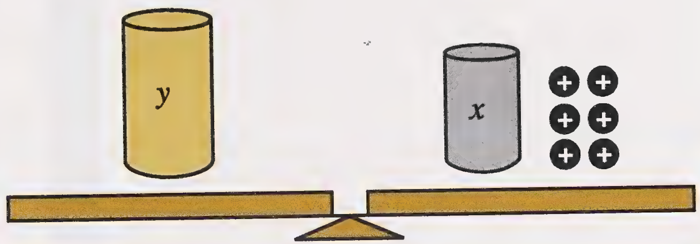
How do you solve an equation with more than one variable?

Example

Solve the equation $y = x + 6$.

Solution

Model the equation.

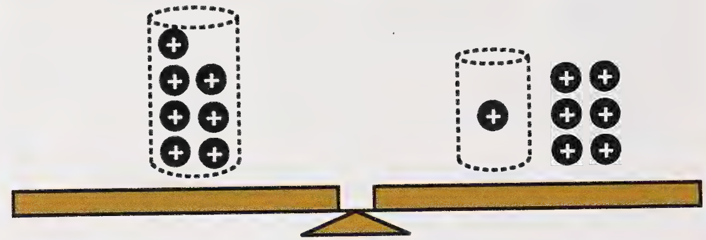


Try to think of values for x and y that will make the equation a true statement.

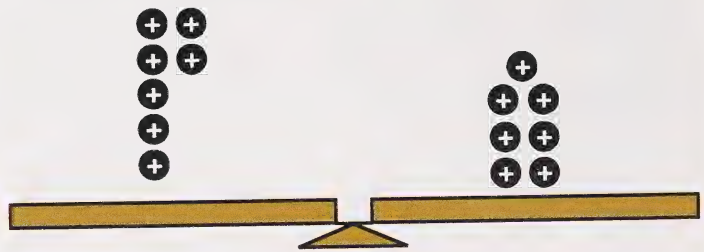
What number is the same as what other number plus 6?

$7 = \boxed{1} + 6$

Verify the solution by replacing the cylinder labelled y with $+7$ and the cylinder labelled x with $+1$.



Now simplify both sides.



The result is $+7$ on each side.

The scale is balanced.

So, $x = 1$ and $y = 7$ is a solution of $y = x + 6$.

The solution can be written as the ordered pair (1,7). The first number in the ordered pair shows the x value and the second number in the ordered pair shows the y value.

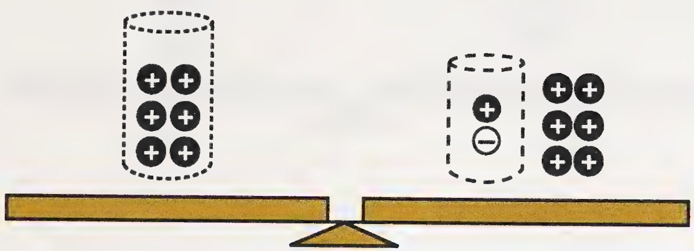
Is (1,7) the only solution of $y = x + 6$?

Try to think of other values for x and y that will make the equation a true statement.

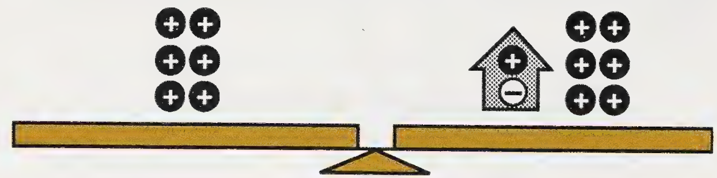
What number is the same as what other number plus 6?

$6 = \boxed{0} + 6$

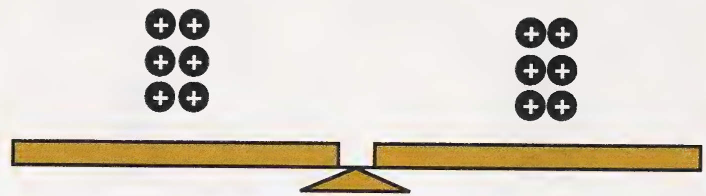
Verify the second solution by replacing the cylinder labelled x with 0 and the cylinder labelled y with + 6.



Now simplify both sides.



The result is + 6 on both sides.



The scale is balanced.

So, $x = 0$ and $y = +6$ or (0,6) is another solution to the equation $y = x + 6$.

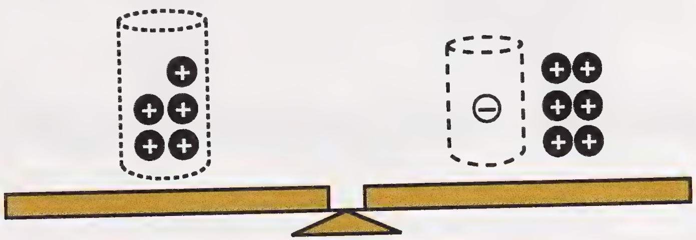
Are there any more solutions?

Try to think of other values for x and y that will make the equation $x + y = 6$ a true statement.

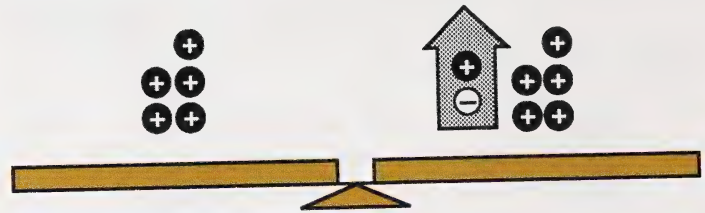
What number is the same as what other number plus 6?

5 = $\boxed{-1}$ + 6

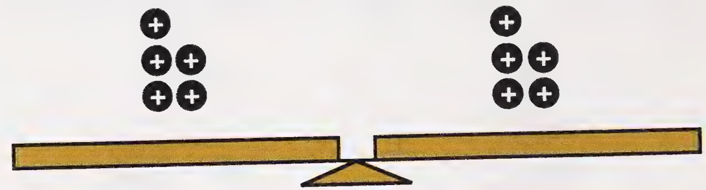
Verify the third solution by replacing the cylinder labelled x with – 1 and the cylinder labelled y with + 5.



Then simplify both sides.



The result is + 5 on each side.



The scale is balanced.

So, $x = -1$ and $y = +5$ or (– 1,5) is another solution.

Introductory Activities

Space for Your Work

For each of the equations do the following.

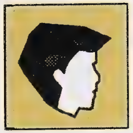
- Use inspection or guess-check-revise methods to find three solutions.
- Write the solutions as ordered pairs.
- Verify the solutions with models.

1. $y = 3x$

2. $y = x - 1$

3. $y = 2x + 1$

See your learning facilitator to check your answers and to receive further instructions.



Working Together

In the Introductory Activities you used inspection or guess-check-revise methods to find some of the solutions of equations with two variables. You verified the solutions using models.

You can also calculate the solutions of equations with two variables using paper and pencil methods.

Example

Solve $y = 2x - 1$.

Solution

Choose a value of x and calculate the corresponding y value.

- If $x = -3$, what is y?

$$\begin{aligned} y &= 2x - 1 \\ &= 2 \times (-3) - 1 \\ &= -6 - 1 \\ &= -7 \end{aligned}$$

So, one solution is $(-3, -7)$.

- If $x = 0$, what is y?

$$\begin{aligned} y &= 2x - 1 \\ &= 2 \times 0 - 1 \\ &= 0 - 1 \\ &= -1 \end{aligned}$$

So, another solution is $(0, -1)$.

- If $x = 5$, what is y?

$$\begin{aligned} y &= 2x - 1 \\ &= 2 \times 5 - 1 \\ &= 10 - 1 \\ &= 9 \end{aligned}$$

So, another solution is $(5,9)$.

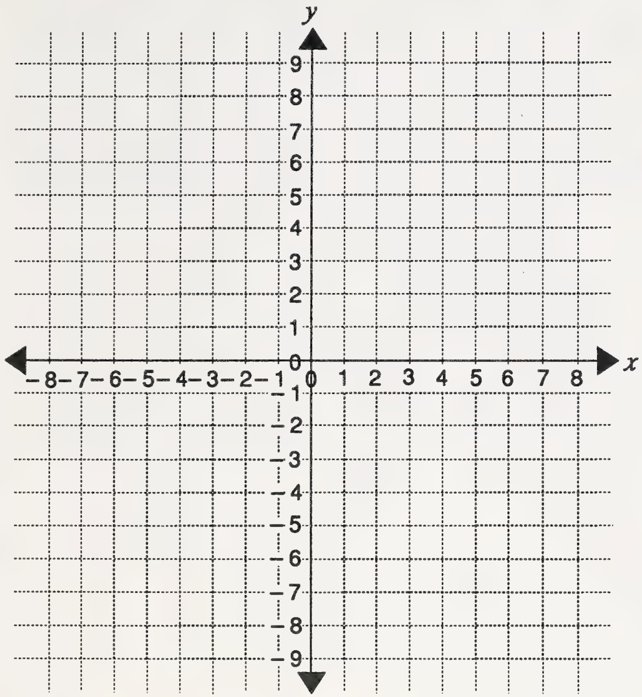
You can continue using this method to find solutions. But it is impossible to list all the solutions as there is an infinite number of them.

You can show the solution on a graph.

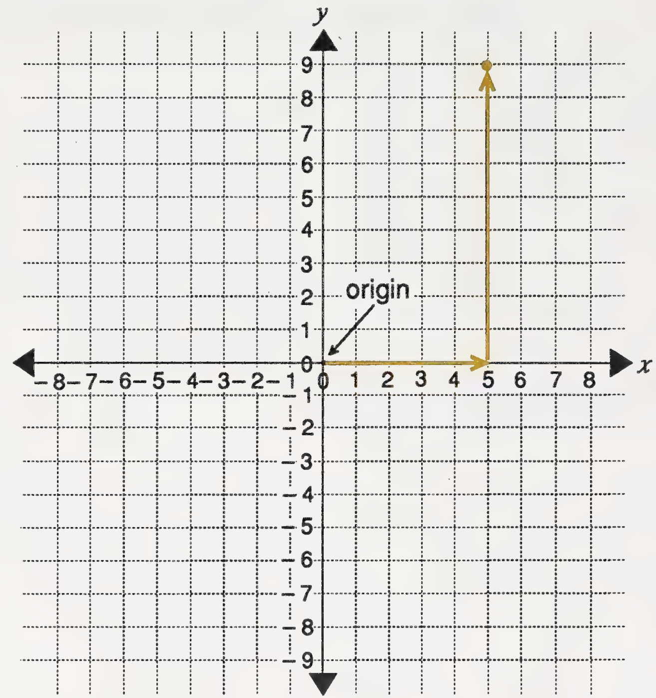
First make a table of values.

x	5	4	3	2	1	0	−1	−2	−3
y	9	7	5	3	1	−1	−3	−5	−7

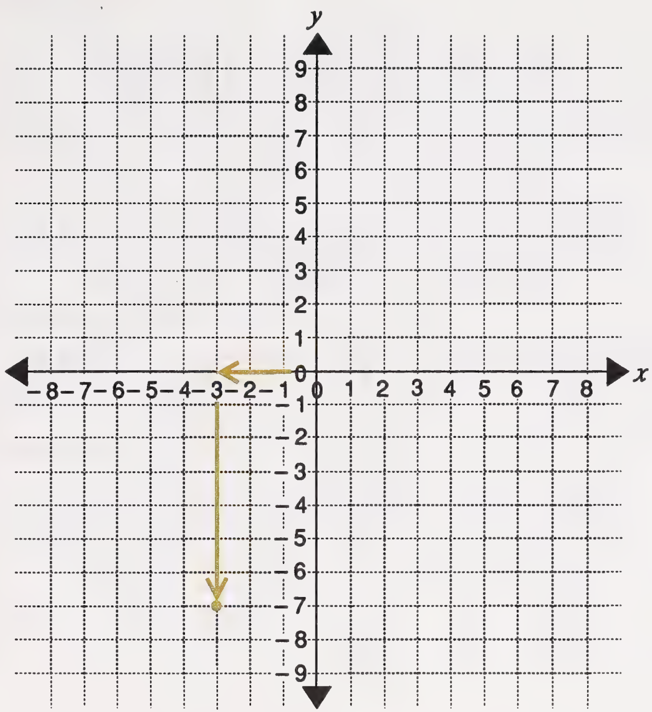
Then label the **horizontal axis** and **vertical axis.**



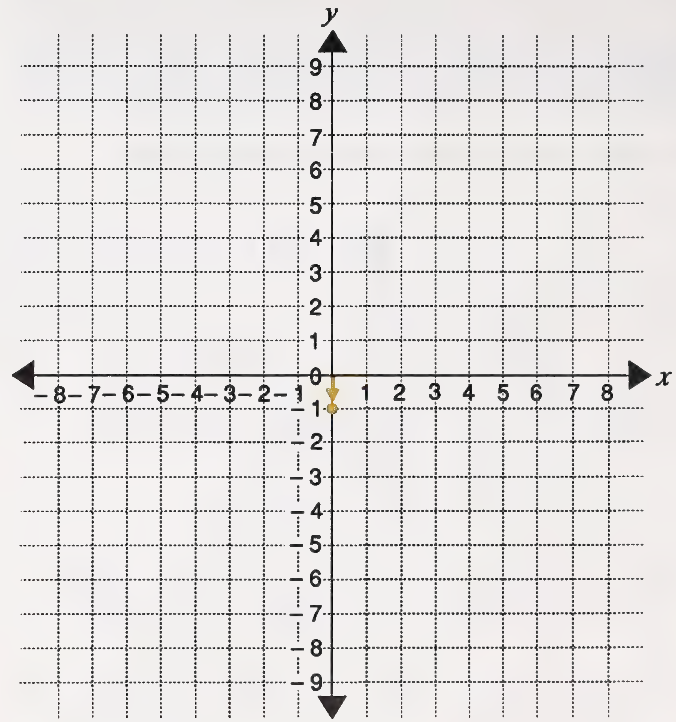
Next plot the ordered pair (5,9). To do this, begin at the **origin** and count right 5 and up 9.



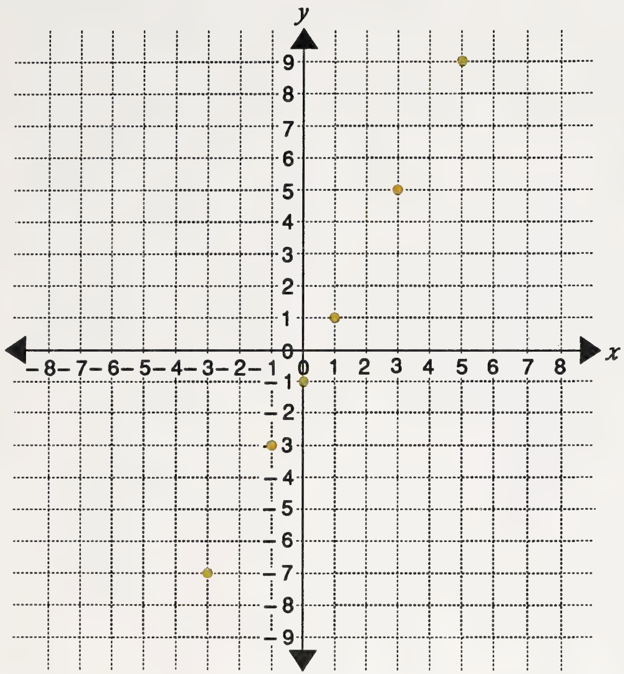
Then plot another ordered pair, such as (– 3,– 7). To do this, begin at the origin and count left 3 and down 7.



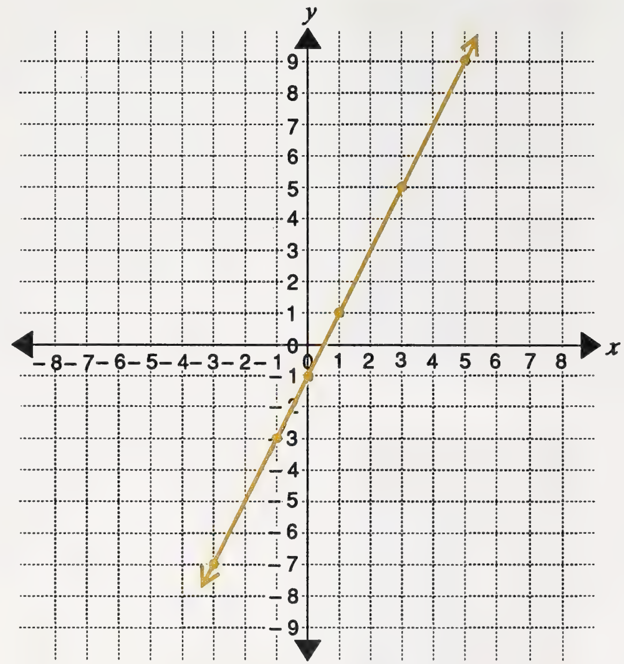
Next plot another ordered pair such as (0,– 1). To do this, begin at the origin and count down 1.



Plot a few more ordered pairs, such as (3,5), (1,1), and (– 1,– 3).



Notice that all the coordinates of ordered pairs are in a straight line. Other solutions will fall in this same line. So, connect the coordinates. Put an arrowhead on each end of the line to show that it continues infinitely.



Practice Activities

Space for Your Work

1. Complete the following tables of values.

a.

$y = 2x$	
x	y
−3	
0	
3	
6	

b.

$y = 3x + 1$	
x	y
−8	
−4	
0	
4	

2. Graph the equations from Question 1.

Space for Your Work

See your learning facilitator to check your answers and to receive further instructions.

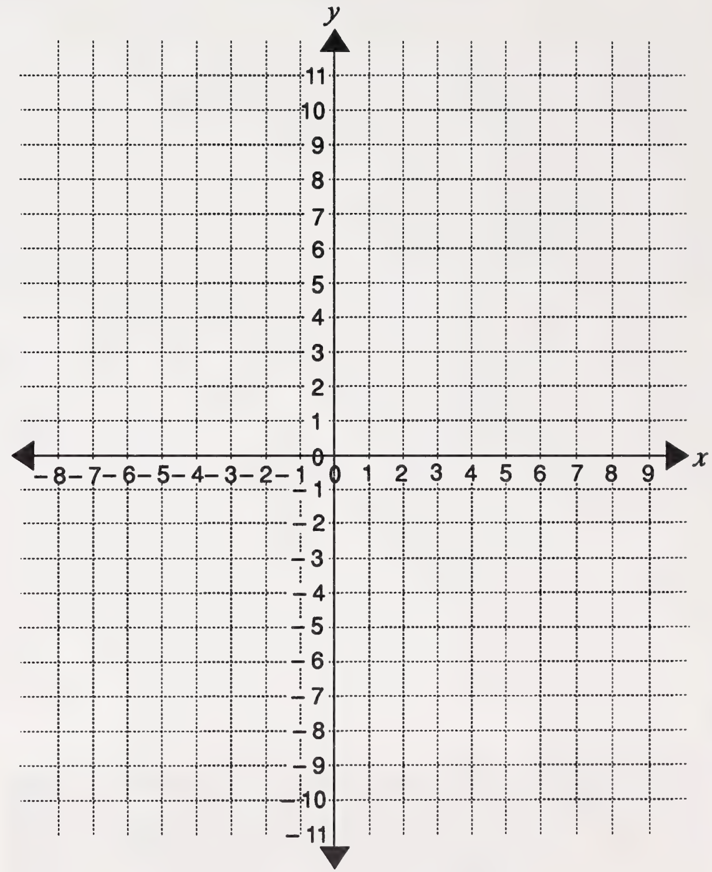
Concluding Activities

1. Find the answer to the riddle by completing the worksheet on the following page.[1]

2. Graph each of the equations from Question 1 on the grid at the right.

3. What do you notice about the graphs of all the equations in Question 1?

Space for Your Work



See your learning facilitator to check your answers and to receive further instructions.

[1] 1982 Creative Publications, Sunnyvale, California 94086 for excerpt from *Algebra With Pizzazz*.

WHY DID ZORNA POUR KETCHUP ON HER BROTHER'S HAND?

Complete the table for each equation. Find each answer in the code key and notice the letter next to it. Write this letter in the box at the bottom of the page that contains the circled number in that row of the table.

$y = -2x$

x	y
1	1
4	2
–5	3
3	4

$y = 2x + 4$

x	y
3	5
–7	6
1	7
–3	8

$y = -3x + 1$

x	y
3	9
–3	10
4	11
–2	12

$y = \frac{1}{2}x - 4$

x	y
10	13
–2	14
4	15
–8	16

$y = -x + 6$

x	y
4	17
–1	18
6	19
0	20

$y = -\frac{3}{2}x - 2$

x	y
4	21
2	22
0	23
–2	24

$y = -3x + 7$

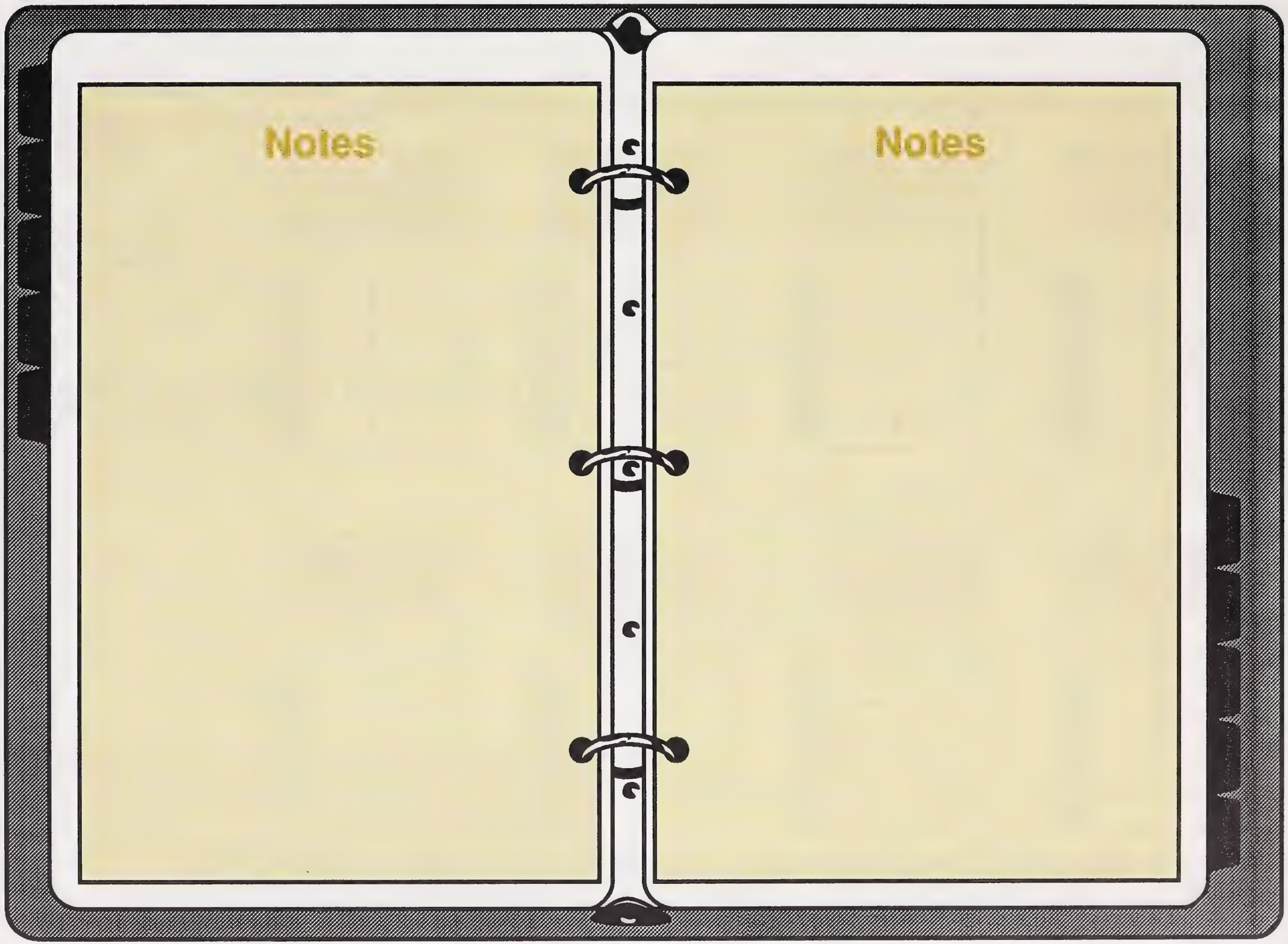
x	y
6	25
1	26
0	27
–2	28

$y = -x + 1$

x	y
–2	29
–9	30
9	31
6	32

CODE KEY	
13	L
10	R
7	A
6	T
4	P
3	M
2	W
1	I
0	N
– 2	H
– 5	D
– 6	B
– 8	E
– 10	O
– 11	S

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32


Notes
Notes

RELATIONS

What Lies Ahead

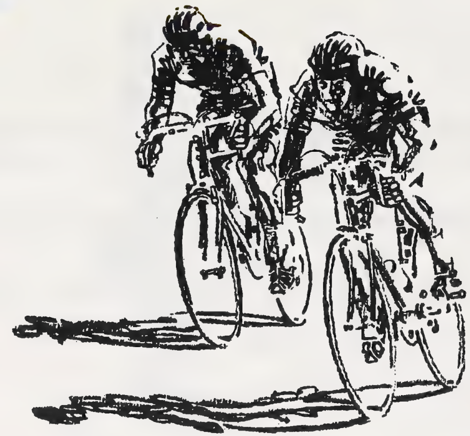
In this section you will learn these skills.

- describing a relation using a table, a rule, ordered pairs, and a graph
- finding missing terms in a sequence

In this section you will use these words.

- relation
- ordered pair
- sequence

Working Together



The picture shows cyclists participating in a race. The time it takes a cyclist to finish the race is related to the distance he covers and the speed at which he travels.

In this section you will learn to describe relations using several methods.

Example 1

Norman and Bob are brothers. When Bob was 7, Norman was 9. When Bob was 8, Norman was 10. How are their ages related?

The relationship can be expressed in several ways.

- The relationship can be shown in a table.

Bob's Age (b)	Relation	Norman's Age (n)
1	1 + 2	3
2	2 + 2	4
3	3 + 2	5
4	4 + 2	6
5	5 + 2	7

- The relationship can be generalized in words.

  Norman is two years older than Bob.

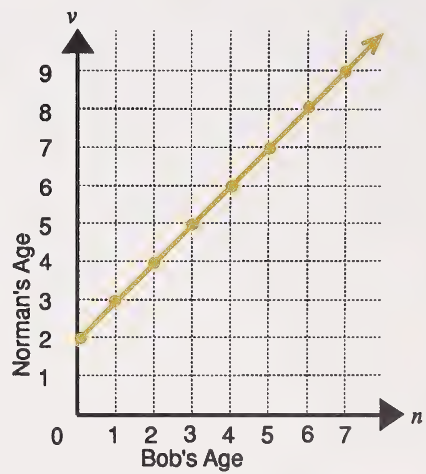
- The relationship can also be described using an equation.

$$n = b + 2$$

- The relationship between Bob's age and Norman's age can be shown by **ordered pairs**.

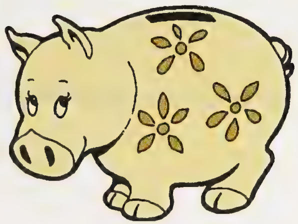
  (1,3), (2,4), (5,7), (6,8), (7,9), ...

- The relationship can be described by a **graph**.



Example 2

A piggy bank contains only dimes. How is the total amount of money in the bank related to the number of dimes?



The relationship can be described in several ways.

- The relationship can be shown in a table.

Number of Dimes (n)	Relation	Value in Cents (v)
1	10×1	10
2	10×2	20
3	10×3	30
4	10×4	40
5	10×5	50

- The relationship can be generalized in words.

  The value in cents is ten times the number of dimes.

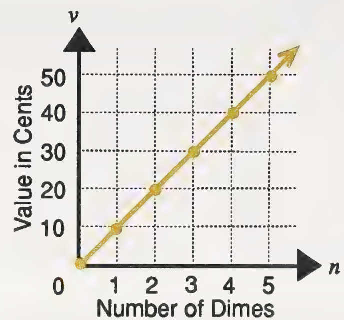
- The relationship can also be described using an equation.

$$v = 10n$$

- The relationship between the number of dimes and value in cents can be described by ordered pairs.

  (0,0), (1,10), (2,20), (3,30), (4,40), (5,50), ...

- The relationship can be described by a graph.



Practice Activities

Space for Your Work

1. Santini likes to go horseback riding. How is the cost related to the riding time?

Riding Time in Hours (t)	Relation	Cost in Dollars (c)
1	$4 + 2 \times 1$	6
2	$4 + 2 \times 2$	8
3	$4 + 2 \times 3$	10
4	$4 + 2 \times 4$	12
5	$4 + 2 \times 5$	14

Describe the relationship by using each of the following methods.

a. Write the words to describe the relation.

b. Write an equation to describe the relation.

c. Write ordered pairs to describe the relation.

Space for Your Work

d. Describe the relationship using a graph.

2. How is Rajah's hourly pay related to Nadia's hourly pay?

Space for Your Work

Nadia's Pay (f)	Relation	Rajah's Pay (g)
5	5 – 1	4
6	6 – 1	5
7	7 – 1	6
8	8 – 1	7
9	9 – 1	8

Describe the relationship several ways by using each of the following methods.

a. Write words to describe the relation.

b. Write an equation to describe the relation.

Space for Your Work

c. Write ordered pairs to describe the relation.

d. Use a graph to describe the relation.

3. How is the length (in metres) of the new room related to the width (in metres) of the new room?

Width in Metres (w)	Relation	Length in Metres (l)
1	$2 \times 1 + 3$	5
2	$2 \times 2 + 3$	7
3	$2 \times 3 + 3$	9
4	$2 \times 4 + 3$	11
5	$2 \times 6 + 3$	13

Describe the relationship by using each of the following methods.

a. Describe the relation using words.

b. Describe the relation using an equation.

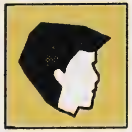
Space for Your Work

c. Describe the relation using ordered pairs.

Space for Your Work

d. Describe the relation using a graph.

See your learning facilitator to check your answers and to receive further instructions.



Working Together

In mathematics a **sequence** is a set of numbers that is ordered according to some rule.

In working with sequences, it is often useful to refer to terms. The first number is called the *first term*, the second number is called the *second term*, and so on.

Example

1st term	2nd term	3rd term	4th term	5th term	
↓	↓	↓	↓	↓	
1,	4,	7,	10,	13	...

What is the fiftieth term in this sequence?

Solution

To find the fiftieth term, try to find a pattern and then apply it.

Term	Number
1	1
2	4
3	7
4	10
5	13

Pattern

+ 3
+ 3
+ 3
+ 3

Each number is three more than the previous number. However, to continue this process to the fiftieth term would be cumbersome.

Examine the relation to generalize the rule algebraically.

Term	Relation	Number
1	1	1
2	1 + 3	4
3	1 + 3 + 3	7
4	1 + 3 + 3 + 3	10
5	1 + 3 + 3 + 3 + 3	13

Then write the relation using multiplication instead of addition.

$$\underset{\text{no threes}}{1} = 1 + 0 \times 3$$

$$1 + \underset{\text{1 three}}{3} = 1 + 1 \times 3$$

$$1 + \underbrace{3 + 3}_{\text{2 threes}} = 1 + 2 \times 3$$

$$1 + \underbrace{3 + 3 + 3}_{\text{3 threes}} = 1 + 3 \times 3$$

$$1 + \underbrace{3 + 3 + 3 + 3}_{\text{4 threes}} = 1 + 4 \times 3$$

Term	Relation	Number
1	$1 + 0 \times 3$	1
2	$1 + 1 \times 3$	4
3	$1 + 2 \times 3$	7
4	$1 + 3 \times 3$	10
5	$1 + 4 \times 3$	13
n	$1 + (n - 1) \times 3$	$1 + (n - 1) \times 3$

You can now use this variable expression to find the fiftieth term.

Evaluate $1 + (n - 1) \times 3$ if $n = 50$.

$$\begin{aligned} 1 + (n - 1) \times 3 &= 1 + (50 - 1) \times 3 \\ &= 1 + 49 \times 3 \\ &= 1 + 147 \\ &= 148 \end{aligned}$$

The fiftieth term is 148.

Concluding Activities

Space for Your Work

1. a. What are the next three terms of the following sequence?

   5, 6, 7, 8, ▩, ▩, ▩

   b. Find the fiftieth term of the above sequence.

2. a. What are the next three terms of the following sequence?

   6, 9, 12, 15, ▩, ▩, ▩

   b. Find the fiftieth term of the above sequence.

3. a. What are the next three terms of the following sequence?

   8, 16, 24, 32, 40, ▩, ▩, ▩

   b. Find the fiftieth term of the above sequence.

See your learning facilitator to check your answers and to receive further instructions.

SUMMARY

What Lies Ahead

In this summary you will review the skills you have learned in this module.

Working Together

Your goal in this module has been to learn skills and words about algebra.

Turn to Section 1 and review the pretest. Correct any errors that you made at the time you did the pretest. You will probably be pleased to discover how much you have learned.

MODULE CONCLUSION

What Lies Ahead

In this section you will complete the Module Assignment.

Working Together

Now that you have studied Module 4 and you have done the required practice, you should be ready for the Module Assignment.

Module Assignment

Turn to the Assignment Booklet and complete the Module Assignment independently. You may refer to your notes, but do not get help from anyone. Afterwards, submit the assignment for a grade and feedback from your teacher.

APPENDIX

GLOSSARY

Additive inverse: the opposite of a number or an expression

$+3$ and -3 are additive inverses.

$5a$ and $-5a$ are additive inverses.

The sum of two additive inverses is zero.

$(+3) + (-3) = 0$

$5a + (-5a) = 0$

Algebra: the branch of mathematics which describes basic arithmetic relations

Algebraic equation: an equation with a variable

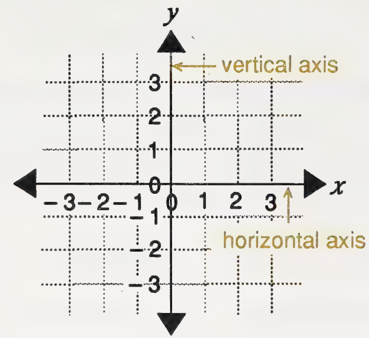
$2a = 6$ is an algebraic equation.

Algebraic expression: a combination of numerals, variables, and/or other mathematical symbols

$5a$ is an algebraic expression.

$3a + 2$ is an algebraic expression.

Axis (plural axes): either of the intersecting number lines of a graph



Cartesian plane: a number grid on a plane

Conditional equation: equations with a variable

$a + 5 = 8$ is a conditional equation.

Coordinates: the two numbers in an ordered pair that locates a point

Coordinate plane: a number grid on a plane

Cross-products: the products that result from cross-multiplying

$\frac{a}{b} \times \frac{c}{d}$

ad and bc are the cross-products.

Equation: a number sentence showing the left-hand side and the right-hand side are equal

$7 + 3 = 2 + 8$ is an equation.

Evaluate: to find the value of an expression

Graph: a pictorial device that displays a relationship

Inequation: a number sentence showing the left-hand side and the right-hand side are not equal

Multiplicative inverse: the reciprocal of a number or an expression

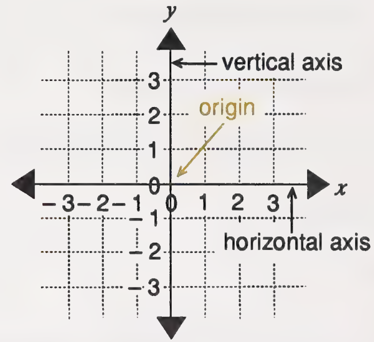
9 and $\frac{1}{9}$ are multiplicative inverses.

a and $\frac{1}{a}$ are multiplicative inverses.

The product of two multiplicative inverses is 1.

Ordered pair: a pair of numbers in which order is important

Origin: the point where the vertical axis and the horizontal axis of a graph intersect



Plot: to locate a point on a grid by means of coordinates

Sequence: a set of numbers that is ordered according to some rule

1, 4, 7, 10, 13, ... is a sequence.

Solving the equation: finding the value of the variable which will make an equation a true statement

Solution: a value of the variable which when used in place of the variable in an equation gives a true statement

$r = 3$ is the solution of $r + 1 = 4$.

Term_1: the product of numerical factors and/or variable factors

$4a$, $2abc$, $3a^2$, $\frac{a}{2}$, and 5 are terms.

Like terms have the same variable factors to the same exponent.

$4a$ and $\frac{a}{2}$ are like terms.

$4a$ and a^2 are unlike terms.

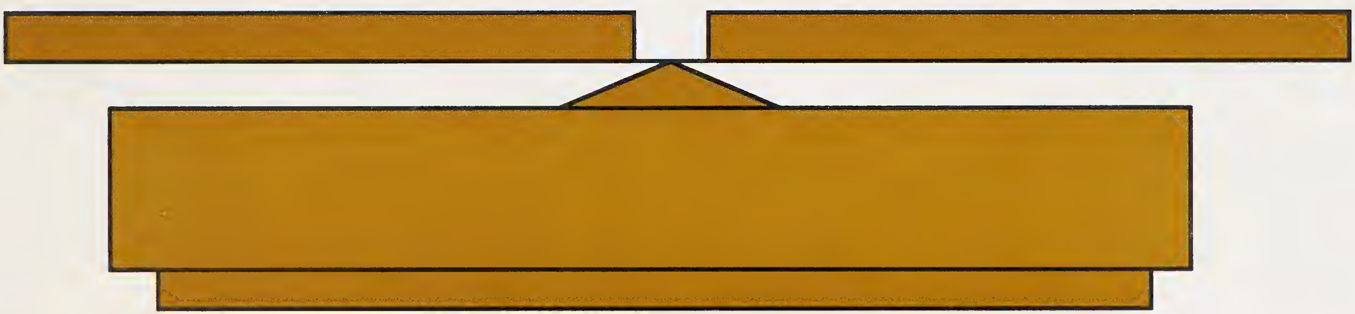
Term_2: the number in a sequence

In the sequence 1, 3, 5, 7, 9, ... the first term is 1 and the second term is 3.

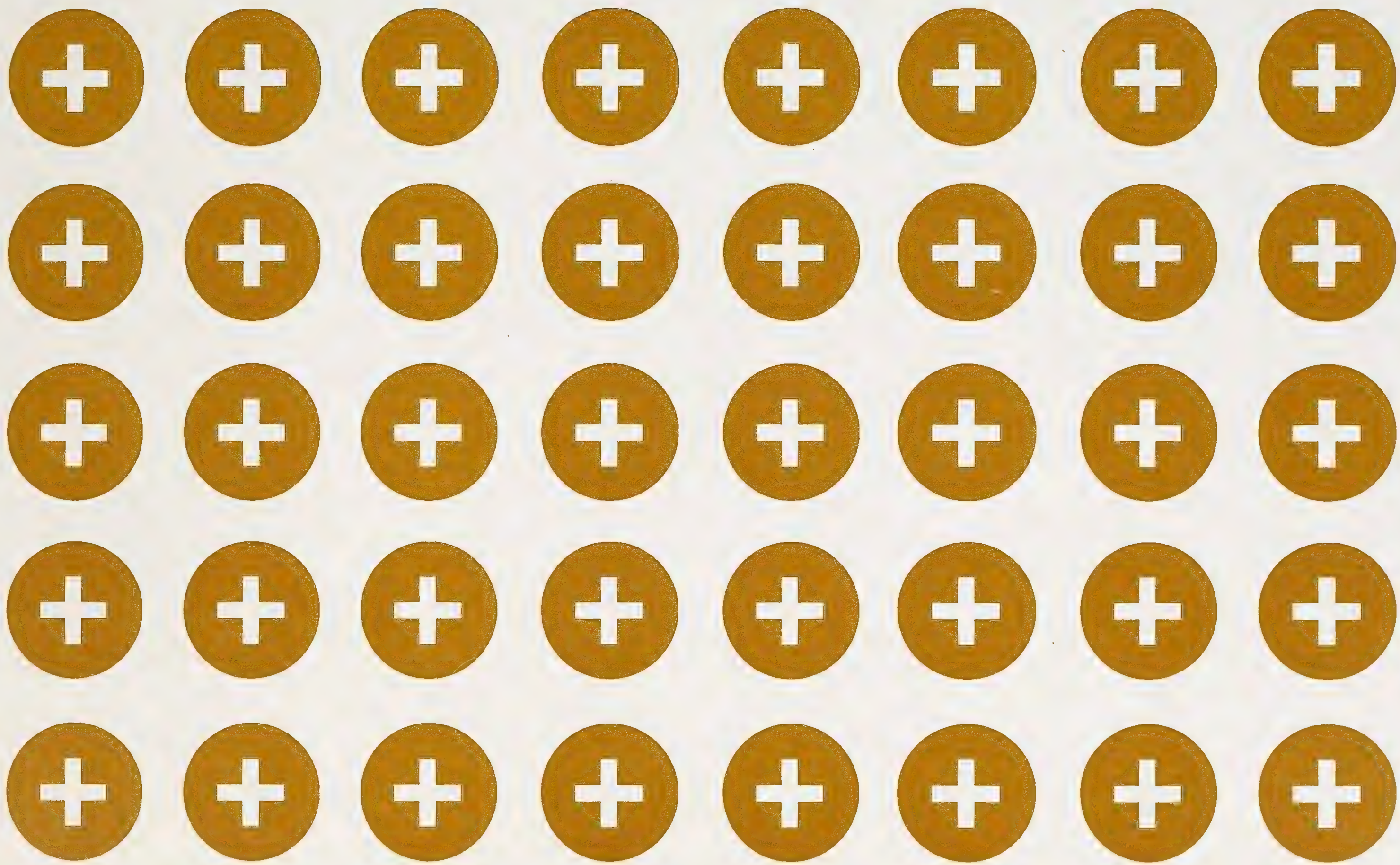
Variable: a symbol (usually a letter) used to represent an unspecified or unknown number

Verifying the equation: checking the solution

EQUATION SCALE



POSITIVE COUNTERS



NEGATIVE COUNTERS

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